Variable Geometry Truss with SMA Wire Actuators (Basic Consideration on Kinematical and Mechanical Characteristics)

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ABSTRACT

The variable geometry truss (VGT) is a truss structural system having a number of lengthadjustable members. It is one of the typical examples of so-called adaptive structure. In the current study, we deal with a wire-actuated VGT having the shape memory alloy wire (SMA wire) for its length-adjustable members. A conceptual design of this type of mechanical system is discussed based on the mechanical characteristics of the SMA wire and the statically indeterminate topology of the wire-actuated VGT. The forward kinematics is dealt with as the static stable state analysis based on the introduced piecewise linear model of the stress-strain relation of SMA. Computational simulation studies of the kinematic motion of the VGT with SMA wire member actuators are conducted; the obtained results demonstrate the feasibility of this type of mechanical system and the significant influence of the SMA wire characteristics on its kinematical behavior.

Keywords: variable geometry truss (VGT), shape memory alloy (SMA), wire member actuators, kinematics, simulation

1. INTRODUCTION

The variable geometry truss (VGT) is a typical example that fits in the concept of adaptive structure. First, we give a brief introduction of VGT. The topics dealt with in this study are summarized.

1-1. Brief overview of VGT researches to date

A variable geometry truss (VGT) is defined as a truss structural system having a number of length-adjustable actuated members [1]; it has the ability to change its geometrical as well as mechanical characteristics to adapt to its given task or to the environment. From the viewpoint of the concept of "structure having actuators", VGT is one of the most typical examples of the adaptive structure [2].

This type of mechanical system has been considered to be an useful instruments for future space missions, say as the space crane [3, 4], robotic manipulator [5], docking mechanism [6] and other promising applications. The authors also have been interested in this mechanical system [7-11]. Figure 1 shows an example of geometrical adaptation motion achieved by a helical-mast type VGT



Figure 1. Motion example of simulated helical-mast VGT in ideal model that reduces disturbance coming from the law of conservation of momentum [11]

taking advantage of its highly redundant kinematical degrees of freedom (DOFs) in order to reduce the disturbance coming from the law of conservation of momentum in space, based on our previous study [11]. This simulation example well represents its expected versatility for various applications. VGT is also considered to be a promising candidate of the basic mechanism of variable geometry structures such as the morphing wing of airplane.

Many of the VGT researches, however, have been still remaining in a conceptual or simulation stage especially in the case of 3D VGT, although some experimental systems are fabricated and demonstrated so far [1, 5, 7, 12]. This is considered to be mainly because the cost and the weight of the actuated length-adjustable truss members; that is, the conventional prismatic actuators based on the ball-screw mechanism or those based on the hydraulic or pneumatic telescopic cylinder are of relatively high cost and heavy weight. From this viewpoint, it is important to devise some lengthadjustable actuators that are of relatively low cost and light weight, in order to realize a VGT as a practical mechanical system.

1-2. Wire-actuated VGT

The authors have been studying another type of VGT, that is, the wire-actuated VGT [13-19]. A wire truss member adopted as length-adjustable actuator is expected to have the advantages of light weight, large stroke, simple actuation, and so on. However, unlike the conventional VGTs of statically determinate topology, a wire-actuated VGT must have a statically indeterminate topology in order to be stable under various mechanical conditions, because of the conspicuous mechanical characteristics of the wire member that it has no rigidity against any compressive force.

We have reported, so far, about the kinematics and the topology design [13, 14], the dynamics [15], a practical setup of 2D experimental system [18], an adaptation motion taking advantage of the distinctive characteristics of wire members [19], and so on. The experimental system of 2D wire-actuated VGT demonstrates the large stroke and easy actuation. However, the placement of the winding on/off actuators as well as the pulleys used to weave the wire members over the truss system is expected to be complicated in the case of 3D wire-actuated VGT, because we have to take account of the direction change of their rotation axes caused by the geometry adaptation motion.

1-3. SMA wire as length-adjustable members

On the basis of the above-mentioned circumstances, we discuss the application of SMA (Shape Memory Alloy) wire to the length-adjustable actuator of the VGT. The SMA wire has the striking capability to change its length by itself; that is, the winding on/off mechanism and the pulleys are not necessary for this type of wire-actuated VGT. In order to realize the VGT with SMA wire actuators, we have to take account of their nonlinear and hysteretic characteristics. In this study, we introduce a piecewise linear model of the SMA characteristics and formulate the kinematic problem as a static stable state analysis. Simulation study of a VGT having SMA wire members is conducted and the feasibility of this type of mechanical system is discussed.

2. ADOPTING SMA WIRE AS ACTUATORS FOR VGT

2-1. Shape Memory Alloy

Shape memory alloy (SMA) is a material that exhibit the shape memory effect, that is, regaining its undeformed original shape when heated, after its deformation performed at a low temperature.

SMA has two crystalline phases called Martensite and Austenite. In the Martensite phase at a low temperature, the material yielding stress is low and the shape of an object made of SMA can be plastically deformed with a relatively small applied force. Heating of the SMA object makes the phase transformation from the Martensite to the Austenite. In the Austenite phase, the SMA object recovers the deformation and returns to its original shape. The SMA object is able to generate a larger force in the recovering process than that applied to cause the plastic deformation in the Martensite phase.

Heating of SMA can be performed externally or internally. It should be noted that the electrical heating by the Joule effect is available as a means for internal heating, since SMA is a conductive material. Cooling of SMA is generally performed by natural air convection.

2-2. SMA Wire and Its Use as Actuator

SMA wire can be plastically elongated by a relatively small tensile force in the Martensite phase at room temperature and contracts toward its original length generating a relatively large contracting force in the Austenite phase at high temperature. Actuators adopting SMA wire have been widely studied so far. However, SMA wire can generate its recovering force only in one direction in most cases, that is, only in its contracting direction. In order to realize bi-directional actuation with



Figure 2. A typical actuating device with SMA wire having bias spring



(Bi-directional actuation taking advantage of statically indeterminate topology)

SMA wire, a kind of bias approach can be adopted [20, 21]. Figure 2 shows a typical bi-directional actuating device with SMA wire by means of a bias spring. The bias spring pulls the actuation point at room temperature, while the SMA wire pulls the actuation point at high temperature. Another type of bias approach is also possible; for example, the approach studied by Dunlop and Garcia [20] uses a bow as a bias spring device.

2-3. Taking Advantage of Statically Indeterminate Topology

A variable geometry truss with wire member actuators intrinsically has a statically indeterminate topology [14]. In the current study, the internal force coming from this statical indeterminacy is taken advantage for the bi-directional length change of SMA wire actuators, each of which itself is a unidirectional actuating device.

Figure 3 shows the conceptual illustration of the bi-directional motion of statically indeterminate VGT with SMA wire members. Each of the two diagonal SMA wire members in the VGT unit is able to exert a tensile force to the other to be elongated, by its contracting force in the heated Austenite phase. In this case, the SMA wire member to be elongated is assumed to be in the Martensite phase.

In order to realize the VGT of this type, one of the most important issues is to guarantee the magnitude of the tensile force to be exerted on the SMA wire member in the Martensite phase to be elongated. The difference in the yielding stress between the Martensite and Austenite phases is the fundamental driving force.

3. MECHANICAL CHARACTERISTICS MODEL OF SMA WIRE

In order to deal with the behavior of the VGT with SMA wire members, we need a suitable model of its mechanical characteristics. As already mentioned above, SMA has two crystalline phases, that is, the Martensite phase at low temperature and the Austenite phase at high temperature. We also have to take account of their intermediate phase in case; however, for the sake of simplicity, we assume each of the SMA wire members is entirely in the Martensite or in the Austenite phase, in this study.

Figure 4(a) shows a typical stress-strain relation of SMA wire. In the Martensite phase, the yielding stress is relatively low and the plastic strain remains after unloading. There is also a stress-



(a) Typical stress-strain relation
 (b) Piecewise linear model adopted in this study
 Figure 4. Mechanical characteristics of SMA wire

strain relation corresponding to the detwinning process at the large strain region. In the Austenite phase, the yielding stress is relatively high and the plastic strain does not remain after unloading. There is a hysteresis between the loading and unloading stress-strain relation.

Figure 4(b) shows the adopted piecewise linear model of the stress-strain relation of SMA wire. The model takes account of the different yielding stress values between the Martensite and Austenite phases, the remaining plastic strain and the detwinning process in the Martensite phase and the hysteretic characteristics in the Austenite phase. On the basis of the adopted piecewise linear model, the following parameters determine the stress-strain relation of the SMA wire: the yielding stress σ_y^m , the yielding strain ϵ_y^m and the detwinning strain ϵ_d^m in the Martensite phase, and the yielding stress σ_y^a , the yielding strain ϵ_y^a and the recovery stress σ_r^a in the Austenite phase.

In the Austenite phase, the calculation of stress value σ corresponding to strain value ϵ is performed in terms of the loading stress-strain relation

$$\sigma = \sigma^a_+(\epsilon) \quad (\dot{\epsilon} > 0) \tag{1}$$

and the unloading stress-strain relation

$$\sigma = \sigma_{-}^{a}(\epsilon) \quad (\dot{\epsilon} < 0) \tag{2}$$

that are obtained based on the piecewise linear characteristic relation given as Fig.4(b). The intermediate stress-strain relation between the characteristic curves $\sigma_+^a(\epsilon)$ and $\sigma_-^a(\epsilon)$ in the unloading process is also assumed as linear with the stiffness value $E_r^a = E_i^a = \sigma_y^a/\epsilon_y^a$, that is the value same as the initial stiffness. The calculation of stress value in the Martensite phase is performed in the similar manner based on the unloading stress-strain relation

$$\sigma = \sigma_{-}^{m}(\epsilon) = 0 \quad (\dot{\epsilon} < 0). \tag{3}$$

4. FORWARD KINEMATICS

Kinematics of VGT is generally discussed in terms of the member lengths vector and the nodal positions vector [7]; the forward kinematic problem is to calculate the nodal positions based on the given actuated member lengths. In the case of the VGT with SMA wire member actuators, the member lengths cannot be controlled in an immediate manner; only the crystalline phase of the SMA wire members can be changed by means of the heating and cooling process.

The forward kinematic problem in this study is formulated as the static stable state analysis that determines the nodal positions based on the external nodal force and the specified crystalline phase of the SMA wire members. We introduce the binary state vector $\boldsymbol{C} = [c_1, \dots, c_{N_W}]^T$ that denotes the crystalline phase of the SMA wire members as

$$c_i = \begin{cases} 0\\1 \end{cases} \text{ SMA wire member } i \text{ is in } \begin{cases} \text{Martensite}\\ \text{Austenite} \end{cases} \text{ phase,}$$
(4)

where N_W is the number of SMA wire members. The nodal positions vector and the nodal force vector are denoted as $\boldsymbol{X} = [\boldsymbol{x}_1^T, \dots, \boldsymbol{x}_{N_X}^T]^T$ and $\boldsymbol{F} = [\boldsymbol{f}_1^T, \dots, \boldsymbol{f}_{N_X}^T]^T$, where N_X is the number of truss nodes. The static stable state analysis is formulated as the potential minimization problem as follows:

Minimize
$$U = V(\mathbf{R}(\mathbf{X}), \mathbf{C}, \mathbf{S}) - \mathbf{F}^T \mathbf{X}$$
 with respect to \mathbf{X}
subject to $\mathbf{L}^R(\mathbf{X}) = \underline{\mathbf{L}}^R$ (5)

where U is the potential energy of the entire system and V is the strain energy of the SMA wire members, that is,

$$V(\boldsymbol{R}, \boldsymbol{C}, \boldsymbol{S}) = \sum_{i=1}^{N_W} v_i(r_i, c_i, s_i)$$
(6)

and v_i is the strain energy of SMA wire member *i*. The vector $\mathbf{R} = [r_1, \dots, r_{N_W}]^T$ denotes the elongation of the SMA wire members. In order to specify the deformation status of the SMA wire members, the residual plastic deformation vector $\mathbf{S} = [s_1, \dots, s_{N_W}]^T$ is also adopted. The equality constraint in the above minimization problem is for the constant rigid member lengths $\underline{\mathbf{L}}^R = [\underline{l}_1^R, \dots, \underline{l}_{N_R}^R]^T$, where N_R is the number of rigid members.

 N_R is the number of rigid members. The geometrical relation between the nodal positions X and the member lengths $L = [l_1^W, \dots, l_{N_W}^W, l_1^R, \dots, l_{N_R}^R]^T$ can be obtained in the following form:

$$\boldsymbol{L} = \begin{bmatrix} \boldsymbol{L}^{W} \\ \boldsymbol{L}^{R} \end{bmatrix} = \boldsymbol{L}(\boldsymbol{X}), \qquad \boldsymbol{L}^{W} = \boldsymbol{B}^{W}\boldsymbol{L}(\boldsymbol{X}), \qquad \boldsymbol{L}^{R} = \boldsymbol{B}^{R}\boldsymbol{L}(\boldsymbol{X}), \tag{7}$$

where B^W and B^R are the boolean matrices for the wire and rigid members. The elongation vector of SMA wire members is expressed as

$$\boldsymbol{R} = \boldsymbol{L}^{W} - \underline{\boldsymbol{L}}^{W} = \boldsymbol{B}^{W} \boldsymbol{L}(\boldsymbol{X}) - \underline{\boldsymbol{L}}^{W}, \qquad (8)$$

where \underline{L}^{W} is the undeformed original lengths vector of the SMA wire members.

We use the gradient projection method to solve the potential minimization problem (5). The gradient of the potential is obtained as

$$\frac{\partial U}{\partial \boldsymbol{X}} = \frac{\partial V}{\partial \boldsymbol{R}} \frac{\partial \boldsymbol{R}}{\partial \boldsymbol{X}} - \boldsymbol{F} = \boldsymbol{H}^T \boldsymbol{B}^W \frac{\partial \boldsymbol{L}}{\partial \boldsymbol{X}} - \boldsymbol{F}$$
(9)

where $\boldsymbol{H}^T = \frac{\partial V}{\partial \boldsymbol{R}} = \begin{bmatrix} \frac{\partial v_1}{\partial r_1}, \cdots, \frac{\partial v_{N_W}}{\partial r_{N_W}} \end{bmatrix}$ and $\frac{\partial v_i}{\partial r_i} = h_i(r_i, c_i, s_i)$ is the tensile force of the *i*th SMA wire member that can be calculated based on the piecewise linear relation introduced in the previous section. On the basis of the gradient projection for rigid member lengths constraint, the allowable nodal position change that decrease the potential is obtained as

$$\Delta \boldsymbol{X} = \gamma \left[\left(\frac{\partial \boldsymbol{L}^{R}}{\partial \boldsymbol{X}} \right)^{+} \frac{\partial \boldsymbol{L}^{R}}{\partial \boldsymbol{X}} - \boldsymbol{I} \right] \frac{\partial U}{\partial \boldsymbol{X}}$$
(10)



Figure 5. Upright initial configuration and basic specification of simulated VGT with SMA wire members

with an adequate small coefficient γ , where $(\cdot)^+$ expresses the Moore-Penrose generalized inverse. The static stable state analysis based on the minimization of the potential is performed by the repetition of the nodal positions modification

$$\boldsymbol{X} \leftarrow \boldsymbol{X} + \Delta \boldsymbol{X};$$
 (11)

accordingly, the update of the state variable S and the Jacobian matrix $\frac{\partial L}{\partial X}$ has to be performed repetitively.

5. FEASIBILITY EXAMINATION BY MEANS OF SIMULATION STUDIES

On the basis of the forward kinematics dealt with in the previous section, static motion of a VGT with SMA wire member actuators is simulated. We study the influence of the interference due to the statically indeterminate topology and the deformation caused by the external force. The feasibility of this type of mechanical system is discussed from the kinematical and static points of view.

5-1. Adopted VGT with SMA wire members

The adopted VGT with SMA wire members is shown in Fig.5 with its basic specification. It has 12 SMA wire members; however, because of the statically indeterminate topology, its kinematical DOFs is not 12 but 6. The upright position shown in the figure is adopted as the initial configuration of the following simulation. Table 1 shows the mechanical characteristics of the SMA adopted for the actuated wire members, based on the piecewise linear model shown in Fig.4(b). The difference of 100MPa between the yielding stress $\sigma_y^m = 200$ MPa in the Martensite phase and the recovery stress $\sigma_r^a = 300$ MPa in the Austenite phase is the primal driving force of this type of mechanical system

Table 1. Mechanical characteristics of SMA adopted in the calculationbased on the piecewise linear model shown in Fig.4(b)

Martensite phase			Austenite phase		
ϵ_y^m	yielding strain	1%	ϵ^a_y	yielding strain	1%
σ_y^m	yielding stress	200MPa	σ_y^a	yielding stress	600MPa
E_i^m	initial stiffness	20GPa	E_i^a	initial stiffness	60GPa
ϵ_d^m	detwinning strain	5%	σ_r^a	recovery stress	300MPa



Figure 6. Simulated static motion of VGT with SMA wire member actuators

based on the statically indeterminate configuration. The SMA wire is able to operate as an actuator that generates contracting force only in its recovery process from elongated state in the Austenite phase. Accordingly, the initial elongation of 3% is assumed for all of the SMA wire members at the upright configuration shown in Fig.5.

5-2. Discussion with obtained simulation result

Figure 6 shows the obtained configurations of the simulated static motion of the 2bay VGT with SMA wire members. The figure also includes the displacement of the tip node in the attained configuration from its initial position shown in Fig.6(0), that is adopted as the workspace vector $\boldsymbol{W} = [w_x, w_y, w_z]^T$. In the figure, the broken and solid thin lines respectively indicate the SMA wire members in the Austenite and Martensite phase and the thick solid lines are the rigid members. As previously mentioned, intermediate state of the SMA wire between the Martensite and Austenite phases is not taken into consideration in the current simulation study.

From the initial configuration (0) that all of the SMA wire members are in the Martensite phase, four diagonal SMA wire members are heated to the Austenite phase and the configuration (1) is attained. This simulation result demonstrates the feasibility of the kinematic motion based on the internal force due to the statical indeterminacy and the difference between the Austenite recovery force and the Martensite yielding load of the SMA wire members. After the four SMA wire members are cooled to the Martensite phase, the configuration (2) is attained. This result indicates that the shift in the attained position can occur during the cooling process.

The difference between the configurations (2) and (4) clearly indicates that the attained position depends not only on the crystalline phases of the SMA wire members but also on its previous configuration. We can also see a slight difference between the configurations (1) and (2) and the configurations (5) and (6). These results show the significance to take account of the history dependence in the geometry adaptation motion.

The configurations from (11) to (13) are attained under the rightward horizontal tip nodal force of 200N. The difference between the configurations (10) and (11) as well as configurations (5) and (12) demonstrates the influence of the external force on the attained configuration. This indicates that the external force has to be taken into consideration for the kinematic motion of the VGT with wire member actuators.

6. CONCLUDING REMARKS

In the current study, actuation of statically indeterminate VGT by means of SMA wire members is discussed from the viewpoint of the basic feasibility of the mechanical system of this type. We introduce the piecewise linear model of the mechanical characteristics of SMA wire and formulate the forward kinematic problem as the static stable state analysis. We simulate the static behavior of the VGT with SMA wire member actuators. The results demonstrate the feasibility of the kinematic motion based on the actuation of the SMA wire members. It is, however, also indicated that the mechanical characteristics of SMA such as its history dependence has significant influence on the kinematic behavior of the VGT; these characteristics have to be taken into consideration in the motion planning of this type of mechanical system. The intrinsic binary nature of the SMA wire member actuators is one of the important topics of the kinematics of this type of VGT. The inverse kinematics taking account of this binary nature as well as the influence of the history dependence is left as the future work of the study.

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