**19th International Conference on**
**Adaptive Structures and Technologies**
October 6-9, 2008
Ascona, Switzerland

# Research on Volume Displacement Sensor

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## ABSTRACT

The reduction of structure-borne sound is a persistent problem in acoustics. Since the 1980's, there has been a great deal of research, and plentiful significant gains have been achieved in active structural acoustic control (ASAC) systems, in which only structural actuator/sensors are employed to control the sound radiated by a structure. One of the primary concerns in ASAC systems is to choose the appropriate sensor. Polyvinylidene fluoride (PVDF) strain sensors have attracted more and more attention in recent years in ASAC systems. PVDF sensors are distributively bonded onto the host structure and have the inherent advantage of integrating over their surface area, which leads to potentially more robust implementations when compared to implementations that use accelerometers. The PVDF sensor bonded onto the host structure can be directly integrated into the smart structure for real-time structural-acoustical control. It is well-known that an ASAC system primarily targets low frequency applications, the development of appropriate PVDF sensors used in ASAC should be based on the low frequency mechanisms of the sound radiation from vibrating structures. Since PVDF sensors measure structural vibration information, the designed sensors should take the relationship between the structural response and acoustic response into consideration. Because the sound power is a function of the surface velocity distribution, the different distribution of surface velocity makes different contribution to sound power. For example, for a simply supported plate, the radiation efficiency of odd-odd order structure modes is higher than that of the other order modes. As a result, the reasonable sensor should be designed to mainly detect these well-radiating velocity distributions. It has been widely accepted that the PVDF sensors can be developed based on measurement of the volume displacement/velocity. At low frequencies, the volume displacement/velocity accounts for the majority of the radiated sound power. The cancellation of the volume displacement strategy for an ASAC system has been shown to be very efficient at reducing sound radiation in the low frequency range. In general, the design of PVDF film is based on an orthogonality relationship with the mode shape function of the structure, so that it can be shaped in such a way that its charge output proportional to the volume displacement. This means the mode shape of the structure should be known accurately in advance. For given boundary conditions there are unique PVDF sensor shapes. Any deviation from mode shape may result in large measurement error. To alleviate this problem, this paper presents a new methodology for designing volume displacement sensors using shaped PVDF film on the basis of integral by parts approach. The design method proposed here does not require knowledge of the structural mode shapes of the structures. As shown the PVDF sensor shape is independent of property of the excitation (the type, position and frequency, etc.). Furthermore, the PVDF sensor proposed here is not sensitive to changes in the boundary conditions. For example, for a beam with one clamped end (the other end is arbitrary), its volume displacement can be measured using a fixed shape of PVDF sensor.

Keywords: volume displacement, piezoelectric sensors, integral by parts

## 1. INTRODUCTION

Polyvinylidene fluoride (PVDF) strain sensors have attracted more and more attention in recent years for use with the active structural acoustic control (ASAC) techniques [1], in which only structural actuator/sensors are employed to control the sound radiated by a structure. PVDF films are distributed sensors and thus avoid spatial aliasing problems, they give little loading on light structures, and are easy to cut into desired shapes. The width of a sensor strip can be varied along its length to achieve the required spatial sensitivity. In this way the output signal only requires a suitable amplifier alleviating the need for further signal processing. The PVDF sensor bonded onto the host structure can be directly integrated into smart structure for real-time structural-acoustical control. Since PVDF sensors measure structural vibration information, the sensors should be designed to take the relationship between the structural response and acoustic response into consideration. The reasonable designed sensor should be able to mainly detect the well-radiating vibration distributions. It has been widely accepted that the PVDF sensors can be developed based on the volume displacement/velocity [2 – 9]. At low frequencies, the volume displacement/velocity accounts for the majority of the radiated sound power. The cancellation of the volume displacement strategy for an ASAC system has been shown to be very efficient for reducing sound radiation in the low frequency range. Ref. [2, 3] gave an exhaustive literature survey on the design of volume velocity sensors for beam- and plate- type structures.

The theory used for designing volume displacement sensors with shaped PVDF films has been well developed. However, the shaped PVDF sensor implementation is often difficult because the sensor design is almost exclusively performed using an orthogonality relationship with the structural mode shape function. This means the vibration mode shape of the beam should be known accurately in advance. For given boundary conditions there are unique PVDF sensor shapes.

To alleviate these problems, this paper presents a new methodology for designing shaped PVDF sensors to detect the volume displacement from a vibrating beam. As opposed to the work of previous authors (using orthogonality relationships with the structural mode shape function, referred as the modal approach), the PVDF shape design method proposed here is based on integral by parts approach. The method proposed here does not require knowledge of the structural mode shapes of the beam. Through design of the shape of the PVDF film, the charge output of PVDF sensor is proportional to the volume displacement of the beam. Finally, numerical simulations are conducted to verify the proposed volume displacement PVDF sensors.

## 2. DESIGN VOLUME DISPLACEMENT SENSOR USING INTEGRAL BY PARTS APPROACH

Consider a beam of length *L*, width *b* and thickness *h*, a piezoelectric film of uniform thickness is attached to the upper surface over the entire length of the beam, as shown in Figure 1. The volume displacement *D* is defined as the integral of the displacement $w(x)$ over the surface of the beam, and can be represented as [6]

$$D = b\int_0^L w(x)dx \tag{1}$$

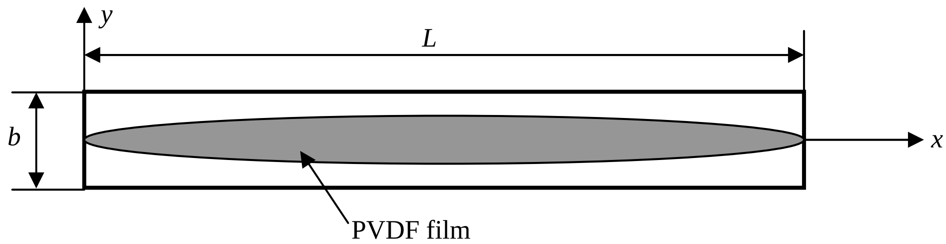


**Figure 1. Beam with a shaped PVDF film.**

As referred to Lee and Moon's work [10], the output charge $Q$ of the shaped PVDF sensor can be expressed as follows,

$$Q = \frac{h + h_f}{2} \int_0^L F(x) \cdot \left( e_{31} \frac{\partial^2 w(x)}{\partial x^2} \right) dx \tag{2}$$

where $h_f$ is the PVDF sensor thickness, $F(x)$ is the shape function of the PVDF sensor, $e_{31}$ is the PVDF sensor stress/charge coefficient.

Integrating Eq. (2) by parts twice, we get

$$Q = \frac{h + h_s}{2} e_{31} \left( F(x) \frac{\partial w(x)}{\partial x} \Bigg|_0^L - \int_0^L \frac{\partial F(x)}{\partial x} \frac{\partial w(x)}{\partial x} dx \right)$$

$$= \frac{h + h_s}{2} e_{31} \left( F(x) \frac{\partial w(x)}{\partial x} \Bigg|_0^L - \frac{\partial F(x)}{\partial x} w(x) \Bigg|_0^L + \int_0^L \frac{\partial^2 F(x)}{\partial x^2} w(x) dx \right) \tag{3}$$

To design shaped PVDF sensors that can accurately measure the volume displacement of the beam, the output charge $Q$ in Eq.(3) should be proportional to the volume displacement $D$. We can assume the PVDF shape function as

$$F(x) = Ax^2 + Bx + C \tag{4}$$

where $A$, $B$ and $C$ are unknown coefficients.

From Eq. (4), we can get $\partial^2 F(x) / \partial x^2 = 2A$, then substituting into Eq. (3),

$$Q = \frac{h + h_s}{2} e_{31} \left( F(x) \frac{\partial w(x)}{\partial x} \Bigg|_0^L - \frac{\partial F(x)}{\partial x} w(x) \Bigg|_0^L + \int_0^L 2Aw(x) dx \right)$$

$$= \frac{h + h_s}{2} e_{31} \left( F(x) \frac{\partial w(x)}{\partial x} \Bigg|_0^L - \frac{\partial F(x)}{\partial x} w(x) \Bigg|_0^L \right) + A \frac{h + h_s}{b} e_{31} D \tag{5}$$

From Eq. (5), it is easy to find that if the PVDF sensor shape function is satisfied,

$$F(x) \frac{\partial w(x)}{\partial x} \Bigg|_0^L - \frac{\partial F(x)}{\partial x} w(x) \Bigg|_0^L \equiv 0 \tag{6}$$

we get $$Q = A\frac{h+h_s}{b}e_{31}D \tag{7}$$

Clearly, if the PVDF shape function $F(x)$ is satisfied in Eq.( 4) and Eq. (6), the output charge $Q$ must be proportional to the volume displacement of the beam. For simplicity, we set $Q = D$ to get,

$$A = \frac{b}{(h+h_s)e_{31}} \tag{8}$$

The next step is to calculate the PVDF shape function coefficients $B$ and $C$ from Eq. (4) and (6). Because Eq.(6) depends on boundary conditions of the beam, the PVDF shape function should be determined for different boundary conditions. It is found that the boundary conditions of the beam can be divided into two groups, for each group of boundary conditions, the volume displacement of the beam can be measured using a fixed shape of PVDF sensor. These two groups of boundary conditions are discussed in the following section.

### 2-1. Beam with one end clamped and the other end having arbitrary boundary conditions

First, we assume that the boundary condition of the beam is clamped at the left end and arbitrary at right end, this boundary condition can be represented as

$$w(0) = \frac{\partial w(0)}{\partial x} = 0 \tag{9}$$

Substituting Eq. (9) into Eq. (6), yields

$$F(L)\frac{\partial w(L)}{\partial x} - \frac{\partial F(L)}{\partial x}w(x) = 0 \tag{10}$$

Clearly, Eq.( 10) can be identically satisfied, only if

$$F(L) = \frac{\partial F(L)}{\partial x} = 0 \tag{11}$$

Using Eq.(11), The coefficient $B$ and $C$ in Eq.(4) can be obtained

$$B = -2AL \text{ and } C = AL^2 \tag{12}$$

Substituting Eq. (12) into Eq. (4), the PVDF shape function can be obtained

$$F(x) = A(x-L)^2 \tag{13}$$

Similarly, for the beam with the clamp at the right end (arbitrary boundary condition at left end), we get $w(L) = \frac{\partial w(L)}{\partial x} = 0$, the PVDF shape function can be represented as

$$F(x) = Ax^2 \tag{14}$$

## 2-2. Beam with zero displacement at the each end

The boundary conditions for zero displacement at the each end can be represented as

$$w(0) = w(L) = 0 \tag{15}$$

Substituting Eq. (15) into Eq. (6), yields $F(L)\frac{\partial w(L)}{\partial x} - F(0)\frac{\partial w(0)}{\partial x} = 0$, which can be satisfied by setting

$$F(L) = F(0) = 0 \tag{16}$$

Substituting Eq. (16) into Eq. (4), the PVDF shape function can be obtained

$$F(x) = A\left(x^2 - Lx\right) \tag{17}$$

If the first derivative of the charge output signal is taken, i.e. the current $I(t)$ is measured, then the sensor output is proportional to the volume velocity of the beam $V$, since $I(t) = \frac{dQ(t)}{dt}$ and $V(t) = \frac{dD(t)}{dt}$.

## 3. NUMERICAL CALCULATIONS

In order to verify the above PVDF volume displacement sensor design method, a beam with dimensions of 500 × 25 × 3.5 mm is considered (over the frequency range of 5Hz to 800Hz). The Young's modulus, density and damping ratio are $2 \times 10^{11}$ Pa, 8700 kg/m$^3$ and 0.01, respectively. The PVDF sensor shapes required for measurement of the volume displacement are shown in Figure 2 and 3 for two groups of boundary conditions, respectively.

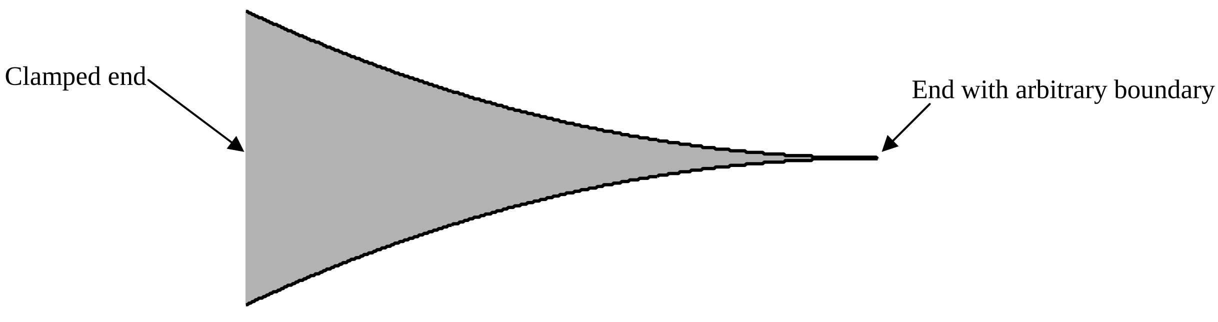


**Figure 2. The PVDF shape for beam with one end clamped and another end arbitrary boundary conditions.**

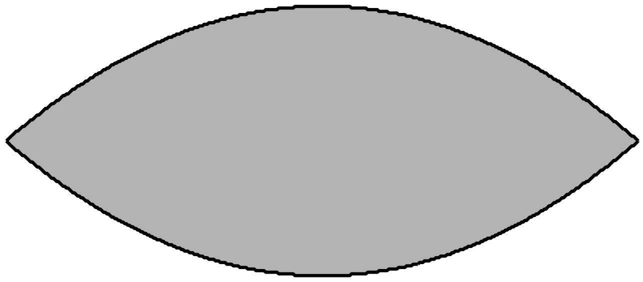

**Figure 3. The PVDF shape for beam with zero displacement at the each end boundary condition.**

To further investigate, Table 1 lists the PVDF shape function $F(x)$ for some typical boundary conditions. From Table 1, it is clearly demonstrated that for clamped-clamped beam, there are many possible sensor shapes to measure the volume displacement, because the PVDF shape function coefficients $B$ and $C$ are arbitrary. The PVDF shapes shown in Figure 2 and 3 can be used to measure the volume displacement of the clamped-clamped beam. Furthermore, it can be found that the PVDF shape for a clamped-free beam (as shown in Figure 2) can be used to measure the volume displacement with the boundary conditions of clamped-clamped, clamped-simply supported or clamped-free.

**Table 1. PVDF shape function $f(x) = Ax^2 + Bx + C$ for some typical boundary conditions**

| | Clamped | Simply supported | Free |
|---|---|---|---|
| Clamped | $B$, $C$ arbitrary | $C = -AL^2 - BL$<br>$B$ arbitrary | $F(x) = A(x-L)^2$ |
| Simply supported | $C = 0$, $B$ arbitrary | $F(x) = A(x^2 - Lx)$ | |
| Free | $F(x) = Ax^2$ | | |

For comparison, the shapes of the PVDF volume displacement sensors designed by the modal approach (see Appendix) are shown in Figure 4. From Figure 4, it can be found that the modal approach yields unique sensor shapes for given boundary conditions. If the boundary condition is changed, the shaped of the PVDF sensor should be modified simultaneously. The main drawback of the modal approach is that the structural mode shape of the beam should be accurately known in advance. Any deviation of the mode shape of the beam may result in large measurement error.

According to above analysis, it can be found that use of the integral by parts approach seems to be more practical than the modal approach. For example, three unique shapes of PVDF sensors (as shown in figure 4.b – d) designed by modal approach are required to measure the volume displacement for the beam with clamped-clamped, clamped-simply supported and clamped-free boundary conditions. However, only one uniform sensor shape (shown in Figure 2) designed by integral by parts can be used to measure the beam with all three of these boundary conditions.

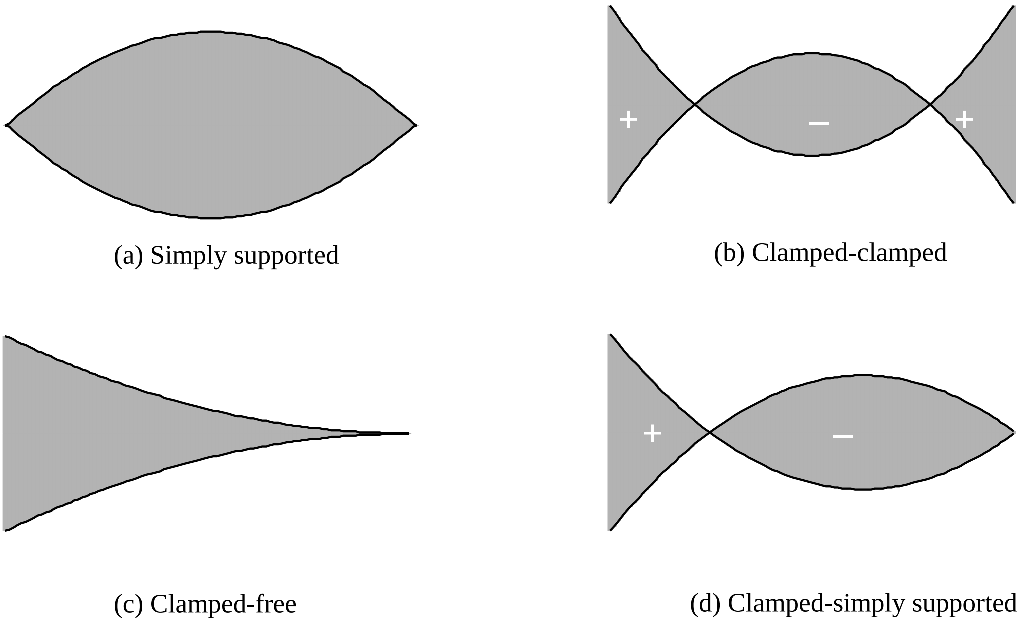


**Figure 4. The shape of the PVDF volume displacement sensors designed by modal approach.**

Assume that the beam is excited by a point force located at $x_d$ = 0.11m. Figure 5 – 7 show the signal outputs of the volume displacement PVDF sensor with the shapes shown in Figure 2 (designed by the integral by parts approach) measuring the beam with clamped-clamped, clamped-simply supported and clamped-free boundary conditions. It can be shown that the charge output curves agree well with analytical volume displacements for different boundary conditions, as expected. It should be noted that only one uniform shape of PVDF sensor is used to measure the volume displacements with these three boundary conditions.

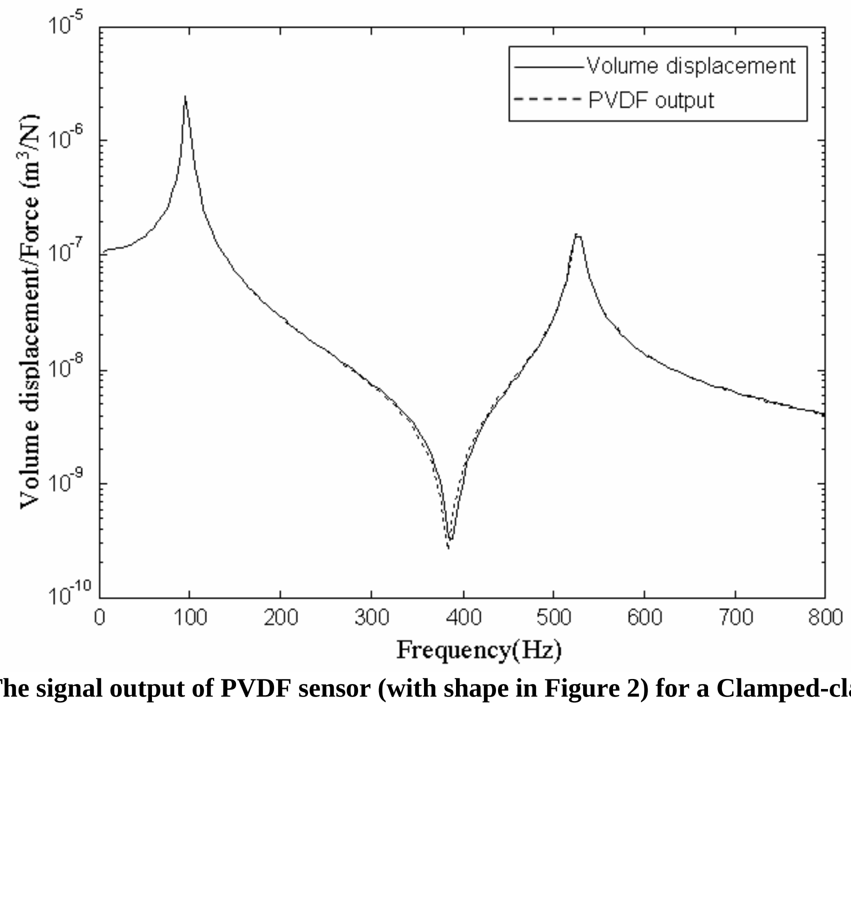


**Figure 5. The signal output of PVDF sensor (with shape in Figure 2) for a Clamped-clamped beam.**

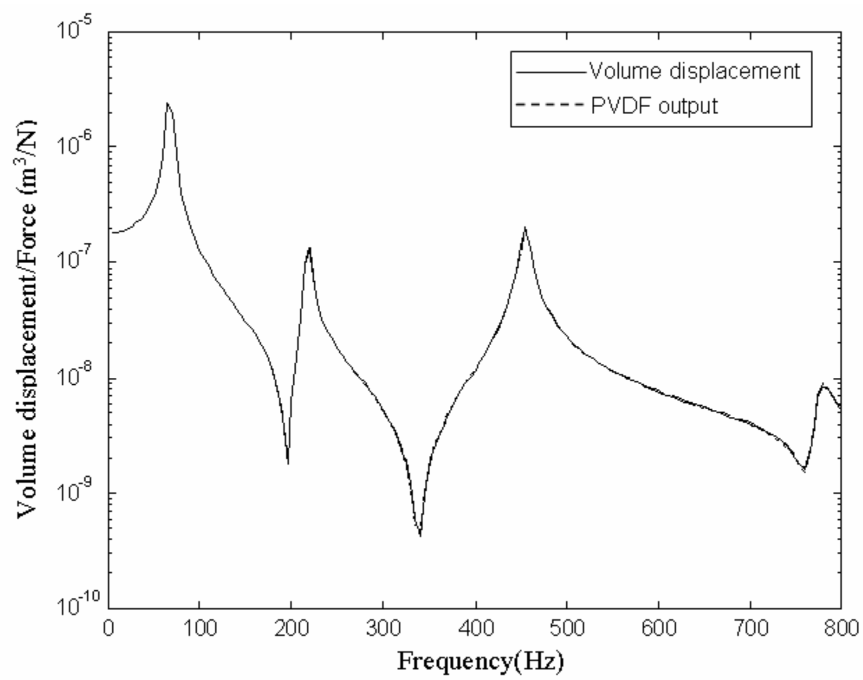


**Figure 6. The signal output of PVDF sensor (with shape in Figure 2) for a Clamped-simply supported beam.**

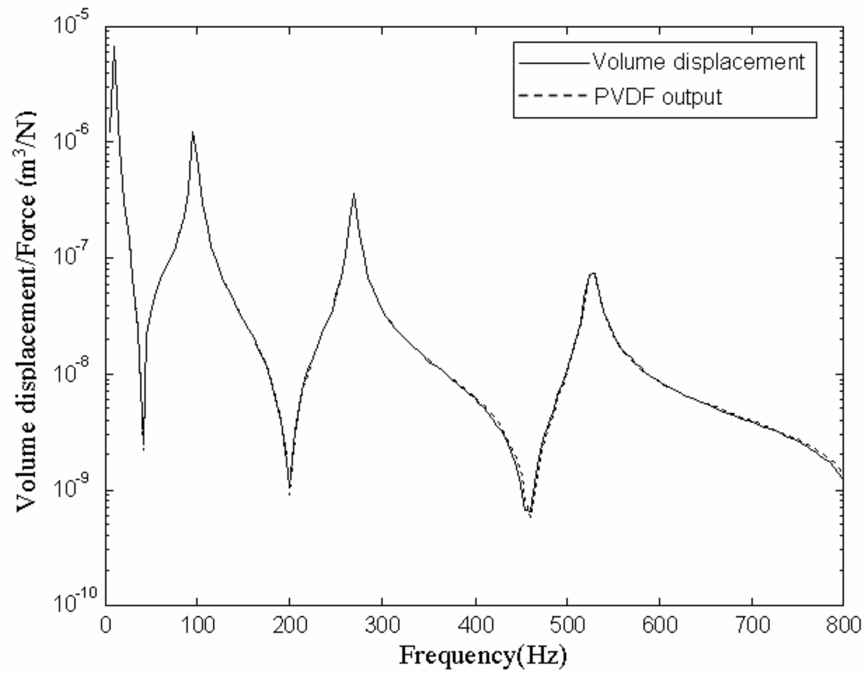


**Figure 7. The signal output of PVDF sensor (with shape in Figure 2) for a Clamped-free beam.**

## 4. CONCLUSIONS

This paper presents a technique for measuring the volume displacement from a vibrating beam using shaped PVDF films on the basis of the integral by parts approach. The main advantage of the method is that the shape function of the PVDF sensor is not sensitive to changes of the boundary conditions. The boundary conditions of vibrating beam can be divided into two groups, that is, (1) one end is clamped and another end is arbitrary; (2) the displacement at each end is zero. As to each group of boundary conditions, the volume displacement of beam can be measured using just a simple fixed shape of PVDF sensor. For example, for a beam with one clamped end, its volume displacement can be measured by using a PVDF sensor with the shape shown in Figure 2, no matter what type of boundary condition is in place at another end. The other advantage is that the PVDF shape designed here is independent of property of the excitation (the type, position and frequency, etc.). The numerical results show the feasibility of this new type of shaped PVDF sensor for the measurement of the volume displacement.

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## APPENDIX. DESIGN SHAPED PVDF VOLUME DISPLACEMENT SENSOR USING MODAL APPROACH

The displacement distribution of a vibrating beam can be represented by a series expansion.

$$w(x) = \sum_{m=1}^{M} A_m \phi_m(x) \tag{A.1}$$

where $A_m$ and $\phi_m(x)$ are the $m$th modal coordinates and structural mode shape function. $M$ is the index for the highest order structural mode considered.

Substituting Eq.(A.1) into Eq.(2), we get

$$Q = \frac{(h+h_s)}{2} e_{31} \int_0^L F(x) \cdot \frac{\partial^2 \left( \sum_{m=1}^{M} A_m \varphi_m(x) \right)}{\partial x^2} dx = \frac{(h+h_s)}{2} e_{31} \sum_{m=1}^{M} A_m \int_0^L F(x) \cdot \frac{\partial^2 \varphi_m(x)}{\partial x^2} dx \tag{A.2}$$

To design shaped PVDF sensor that can accurately measure the volume displacement, the output charge $Q$ in Eq.(A.2) should be proportional to the volume displacement $D$ in Eq.(1). As referred to Lee and Moon's idea [10], the PVDF shape function $F(x)$ in Eq.(A.2) is assumed to be a linear combination of the second derivative of the mode shapes of the vibrating beam, yields

$$F(x) = \sum_{k=1}^{M} B_k \frac{\partial^2 \varphi_k(x)}{\partial x^2} \tag{A.3}$$

where $B_k$ is the unknown coefficient.

Substituting Eq.(A.3) into Eq.(A.2), we get

$$Q = \frac{(h+h_s)}{2} e_{31} \sum_{k=1}^{M} B_k \sum_{m=1}^{M} A_m \int_0^L \frac{\partial^2 \varphi_k(x)}{\partial x^2} \cdot \frac{\partial^2 \varphi_m(x)}{\partial x^2} dx \tag{A.4}$$

According to the orthogonality property of the structural mode shapes,

$$\int_0^L \frac{\partial^2 \varphi_k(x)}{\partial x^2} \cdot \frac{\partial^2 \varphi_m(x)}{\partial x^2} dx = 0 \qquad \text{if} \quad k \neq m \tag{A.5}$$

Using Eq.(A.5), Eq.(4) can be rewritten as

$$Q = \frac{(h+h_s)}{2} e_{31} \sum_{k=1}^{M} B_k A_k \int_0^L \left( \frac{\partial^2 \varphi_k(x)}{\partial x^2} \right)^2 dx \tag{A.6}$$

Substituting Eq.(A.1) into Eq.(1), the volume displacement can be rewritten as

$$D = b \sum_{k=1}^{M} A_k \int_0^L \varphi_k(x) dx \tag{A.7}$$

Compare Eq.(A.7) to Eq.(A.6), assume that $Q = D$, we get the PVDF shape coefficients

$$B_k = -\frac{2b}{(h+h_s)e_{31}} \frac{\int_0^L \varphi_k(x) dx}{\int_0^L \left( \frac{\partial^2 \varphi_k(x)}{\partial x^2} \right)^2 dx} \qquad k = 1, 2, \ldots, M \tag{A.8}$$

From Eq.(A.8), it is easy to find shape coefficients $B_k$. Substituting $B_k$ into Eq.(A.3), the shape function $F(x)$ for the PVDF sensor can be obtained for measurement of the volume displacement.