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The Design of Morphing Aerofoils using Compliant Mechanisms

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ABSTRACT

The focus of this paper is the design of a compliant system that is able to provide structural control and motion to the trailing edge of a morphing aerofoil. An initial skeletal frame type ground structure is selected and then various members are replaced with actuators whose strains are controlled in order to provide a predetermined surface deflection. The objective is defined in two ways, using the distance either to defined surface nodes or to the required surface. A method to choose the optimum actuator locations using a forward subset selection procedure is described, which has been shown to be very efficient compared to an exhaustive search or methods such as a genetic algorithm. These approaches are demonstrated on simulated examples.

Keywords: Morphing; Compliant Mechanism; Aerodynamic Load

1. INTRODUCTION

The design of conventional fixed wing aircraft is constrained by the conflicting requirements of multiple objectives. Mechanisms such as deployable flaps provide the current standard of adaptive aerofoil geometry, although this solution places limitations on maneuverability and efficiency, and produces a design that is non-optimal in many flight regimes. The development of new smart materials together with the always present need for better UAV performance is increasingly prompting designers towards the concept of morphing aircraft [1-4]. These aircraft possess the ability to adapt and optimize their shape to achieve dissimilar, multi-objective mission roles efficiently and effectively. One motivation for such uninhabited aircraft are birds that morph between cruise and attack missions by changing their wing configuration accordingly. Birds also use camber and twist for flight control.

The structural technologies available to achieve shape changes in a morphing aircraft fall into two major categories, namely planform changes using *rigid* mechanisms, and compliance (for example wing twist or compliant mechanisms). Methods using compliance are of interest in this paper, and in the use of compliant mechanisms to deform the camber of the airfoil section. A compliant mechanism is a single piece structure designed to transmit motion and force mechanically relying solely upon elastic deformation of their constituent elements. The design of such mechanisms treads a balance between achieving adequate stiffness in order that external loads may be supported yet simultaneously be flexible enough that the required motion due to applied loads is realized. Various strategies for the design of compliant mechanisms have been developed in past studies however two basic categories may be defined. A kinematics approach replaces flexure joints with conventional pivots and a torsional spring system. This tends to provide a solution with concentrated areas of compliance within the structure, so called lumped compliance. An alternative is to take a more structural view of the problem using topology optimization methods. This continuous optimization problem is formulated by defining a structural element only by the loads it is to carry, its volume (cost) and design requirements such as stress and strain limitations.

The structural systems employed to create the smooth variable camber have ranged from a mechanistic approach, and ranging through a number of patented designs, to the more recent application of compliant and flexible structures. For example, Saggere and Kota [5] described a method of providing small shape changes in structures comprising flexible beam elements. Here the emphasis was placed on the achievement of smooth curved forms by a compliant structure in a two stage procedure: topological synthesis and dimensional synthesis. The problem of topology generation was addressed by specification of a series of output points on the 2D structural surface from which a topological design forms the basis of the dimensional synthesis problem.

Haftka and Adelman [6] considered the problem of locating actuators within space structures for geometric control, and noted that if regarded as a continuous problem then conventional gradient based optimisation methods may be employed. For the discrete actuator location problem the result is an integer programming problem that is typically more computationally expensive to solve than a continuous problem. In this case two iterative improvement techniques are applied, Worst-Out-Best-In (WOBI) and Exhaustive Single Point Substitution (ESPS). The WOBI algorithm begins with an initial configuration containing the desired number of actuators and removes a number of actuators considered to be worst by whatever performance measure is chosen. Their place in the configuration are taken by those locations considered best and the process continues until no further performance benefit is found by substitution. The ESPS algorithm moves each actuator in the initial configuration to each redundant location in turn, analysing the performance at each trial step. The configuration where no further performance increase can be found is finally settled upon. Other approaches to actuator location selection have also been considered [7].

This paper investigates statically and kinematically determinate truss structures as candidates for the formation of a conformal trailing edge. The promise of these structures is that by the introduction of a number of active actuated elements in place of passive truss elements then the structure would deform increasing the elastic energy. An approximation of predefined surface form is created, differentiated from the undeflected form by variation of the aerofoil mean line. Two methods of comparing the target and deflected surface forms are presented defined as geometry comparison and shape comparison methods. In addition a number of methods for locating the optimal placement of the active elements are employed that may be categorised as exhaustive, heuristic and iterative improvement methods. The performance of the selection methods with varying actuator strain constraints, actuator quantity and truss topology is discussed.

2. PROBLEM FORMULATION

For a structure forming the basis of a morphing wing the applied actuation force, $\mathbf{f}_{\mathrm{act}}$, and the applied aerodynamic load, $\mathbf{f}_{\mathrm{aero}}$, are related to the structural displacements, $\mathbf{q}$, by the non-singular stiffness matrix, $\mathbf{K}$. The actuation force is defined as the vector of forces applied to the structure by an array of active elements, and thus $\mathbf{f}_{\mathrm{act}} = \mathbf{Bu}$, where the matrix $\mathbf{B}$ determines the actuator location and the vector $\mathbf{u}$ are the actuator forces. Thus

$$\mathbf{Kq} = \mathbf{f}_{\mathrm{aero}} + \mathbf{Bu} \tag{1}$$

A reduced selection of structural displacements, $\mathbf{q}_{\mathrm{s}} = \mathbf{Cq}$, are of interest for the creation of a conformal trailing edge, where $\mathbf{C}$ is a binary matrix to select degrees of freedom. Then

$$\mathbf{q}_{\mathrm{s}} - \mathbf{C}\mathbf{K}^{-1}\mathbf{f}_{\mathrm{aero}} = \mathbf{C}\mathbf{K}^{-1}\mathbf{B}\mathbf{u}\,. \tag{2}$$

Note that the applied actuation force output must be corrected to overcome the stiffness of the active element.

Two displacement control objectives are considered, denoted as a geometry objective and a shape objective.

2-1. Geometry objective

Considering an initial NACA 0012 aerofoil profile, the trailing edge portion of the profile is defined as the region $0.6c$ to c, where c is the chord length which in this symmetric case is equal to the mean line length. The mean line in this region is defined according a quadratic function. The definition of a conformal trailing edge fixes the mean line and slope at $0.6c$, and thus gives a single degree of freedom related to the amplitude of the deformation. In addition we assume that the profile thickness at each point on the length of the mean line remains equivalent in the initial and target profiles. The vector of target displacements may now be defined as those displacements required to transform the surface joints of the aerofoil to the target profile defined by a non zero value deformation. The geometry objective is thus quantified by calculation of the Sum of Squares Error (SSE_{g}) of $\mathbf{q}_{\mathrm{s}}$ from this target.

2-2. Shape objective

The shape objective forms the target profile in a similar manner to the geometry objective, however in this case the nodal positions that define the target profile are dependent not just on the mean line function but also the location of the surface nodes of the deflected structure. In this case the target nodal displacement is defined as the displacement required to transform a surface node to the nearest point on the corresponding upper or lower surface of the target profile. The shape objective can now be quantified by calculation of the Sum of Squares Error (SSE_{s}) based on the target deflection vector for the shape objective.

3. SUBSET SELECTION

The actuation location problem is essentially one of subset selection, where the optimum candidate columns of **B** are selected. From Equation (2), neglecting the aerodynamic load for illustration, the objective is to choose the best columns of $\mathbf{C}\mathbf{K}^{-1}\mathbf{B}$ that represent $\mathbf{q}_{\mathrm{s}}$.

Four methods were applied that either allow searching of the entire model space, searching of a fraction of the total model space or alternatively provide a systematic method to examine a *path* through the model space.

For problems in which the number of possible solution combinations is suitably small it is possible to implement an Exhaustive Search (ES) procedure for the evaluation of all possible actuator location combinations. Within the confines of each possible subset, a gradient based constrained optimisation routine is employed to evaluate the optimum values of actuator forces. If the search space within all possible subsets is convex then this procedure reaches a global optimum at the expense of high computational cost.

A Genetic Algorithm (GA) is a search method that seeks to improve the selection by applying the principles of natural selection. For similar problems in which the use of an exhaustive search was not feasible then heuristic methods that permit inferior cost moves, such as a GA, were recommended [8]. The limiting features of a GA are the requirement for a large number of objective function evaluations and typically the rapid location of optimal regions within the search space but with slow convergence to an optimum value.

Stepwise Forward Selection (SFS) attempts to provide a path through the search space whereby a single actuator location is added at each step, keeping previously selected locations until a particular termination criterion is met. This may be if insufficient improvement to the model is made by the addition of the variable or the full number of subsets is reached. The column that provides the closest correlation to the current residual vector is selected in turn, based on angles between vectors [9]. The applied actuation force is limited to a range of values dependent upon the real actuator constraints, i.e. the maximum and minimum available strains.

The process of applying the maximum possible regression coefficient or that which results in an orthogonal residual implies a greedy selection process that may overlook possible improved combinations of actuator locations and strains. A number of well established methods are able to provide an improved solution compared to SFS methods. Backward Selection begins with the full model and sequentially removes variables from the starting set until a subset of required size is reached, whilst methods such as WOBI begin with the desired number of variables and sequentially add and subtract variables from the subset. For both methods the application of upper and lower bounds to the regression coefficients proves difficult to implement.

Less aggressive forward selection techniques including the Lasso and Incremental Forward Selection (IFS) algorithms [10, 11] have been developed more recently. IFS is a more cautious version of SFS and at each step the coefficient of the variable most closely correlated with the current residuals is incremented a fixed step. By reducing the step size, improvements in the correlation between the actual and target structure response may be achieved at the cost of an increased computational burden.

4. A 14 ELEMENT TRUSS STRUCTURE

A simple 14 element truss forming the trailing edge between 0.6*c* and *c* (*c* = 100mm) of a NACA 0012 aerofoil is illustrated in Figure 1. Each of the 14 elements provides a possible location for actuator substitution with the goal of replicating either the illustrated shape or geometry objective forms. Both of the objectives are defined by the deformation of the mean line resulting in a trailing edge deflection that would require 10° equivalent flap angle [12] in order to achieve a similar aerodynamic force with a discrete hinge device.

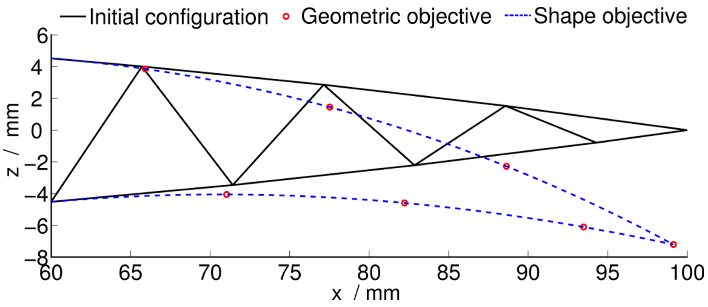

Figure 1. The 14 element trailing edge in its initial configuration, and the target geometry and approximate target shape.

4.1. Geometry objective results

Figure 2 shows the effect of varying the maximum and minimum allowable actuation strain on the geometry objective (SSE_g). Three actuators were located in the structure using 4 methods: Exhaustive Search (ES), Genetic Algorithm (GA), Stepwise Forward Selection (SFS) and Incremental Forward Selection (IFS) and for maximum actuator strains of ε^{max}. The GA provided a match with the optimal ES solution for all values of ε^{max}. For $\varepsilon^{max} = 0.01, 0.025, 0.05$ the search space was constrained by limitations on the maximum absolute actuation strain, however for $\varepsilon^{max} \geq 0.07272$ the search space was constrained only by the number of actuator substitutions. In this instance the minimum obtainable SSE_g was 0.57536, a result illustrated in Figure 3, which gives the target node distribution.

SFS provided the optimum actuator locations and strains when the search space was limited to $\varepsilon^{max} \leq 0.07272$. If ε^{max} was increased beyond this the method became overly greedy, resulting in a large first step. This aggressive selection method bypassed improved selections, resulting in high applied strains in the first selection step instead of a more even actuation distribution that characterises optimal designs. As the search space became larger the results in this case illustrate the worsening performance of the SFS method when compared directly to results obtained with reduced ε^{max}. The conservative progression of the IFS method did not permit the selection of the optimal locations; for $\varepsilon^{max} = 0.05$ it was the worst performing method. However as the size of the search space was increased IFS did not suffer the performance deterioration that afflicted the SFS procedure.

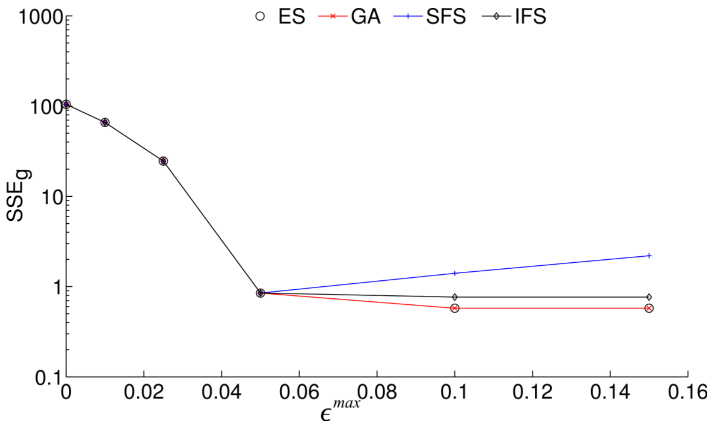

Figure 2. Geometry objective (SSE_g) against maximum absolute actuation strain (ε^{max}) for a 14 element truss with 3 actuator substitutions.

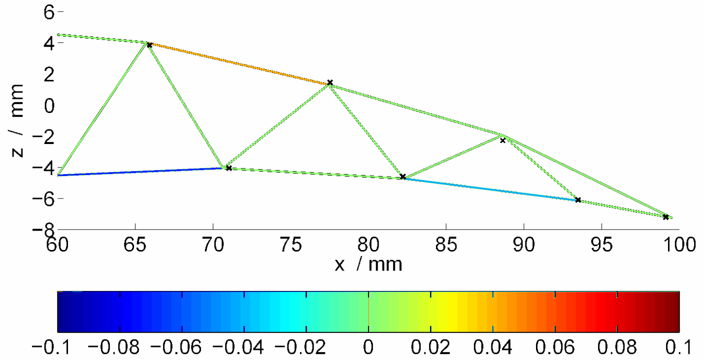

Figure 3. 14 element truss with 3 actuators located using ES and the geometry objective, together with the target node geometry. The element shade refers to the applied strain.

4.2. Shape objective results

Figure 4 show the results for the shape objective SSE_S for the 14 element truss with respect to varying ε^{max}. Three actuator substitutions were again selected using the ES, GA, SFS and IFS methods. As with the geometry objective, the GA results matched those obtained with ES and thus they may be considered to be globally optimal. In the cases where $\varepsilon^{max} < 0.07232$ the search space was constrained by ε^{max}. This provided close agreement with the geometry objective solutions. However for $\varepsilon^{max} \geq 0.07232$ the location selections switched to the elements forming the lower facing surface of the aerofoil illustrated by Figure 5. Comparison of these shape objective results with those derived using the geometric objective reveals a similar trend, whereby in the region $0 \leq \varepsilon^{max} \leq 0.05$ large reductions in the SSEg and SSEs were achievable with small increases in ε^{max}.

The SFS method for $\varepsilon^{max} \leq 0.025$ provided the optimal selection however, as with the geometry objective, as the search space was increased the greedy selection method fell in to a non-optimal path. In each case taking an excessively large first step resulted in highly localised deflections of the structure. The IFS method obtained optimal actuator selections when $\varepsilon^{max} \leq 0.025$, however as $\varepsilon^{max} \geq 0.1$ the method provided optimum locations with non-optimum regression coefficient values.

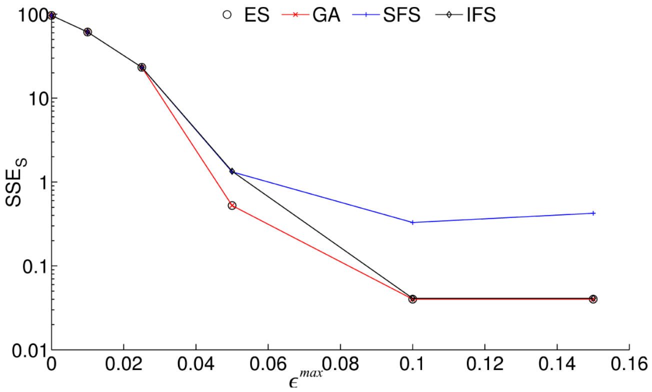

Figure 4. Shape objective (SSE_S) against maximum absolute actuation strain (ε^{max}) for a 14 element truss with 3 actuator substitutions.

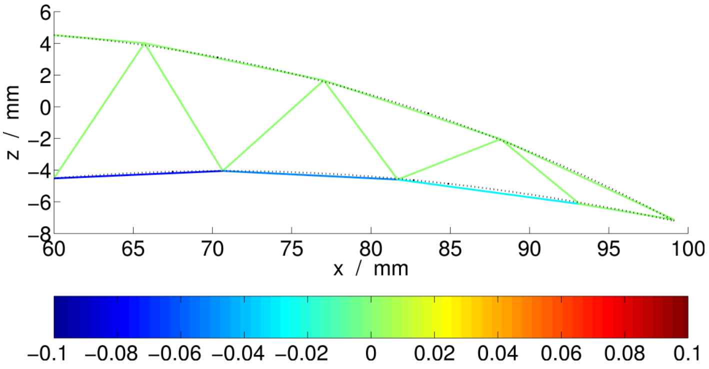

Figure 5. 14 element truss with 3 actuators located using ES and the shape objective, together with the target node geometry. The element shade refers to the applied strain.

5. TWO 1752 ELEMENT TRUSS STRUCTURES

Figure 6 illustrates two statically and kinematically determinate truss structures comprising

1752 elements and referred to as ’A’ and ’B’. Each structure is geometrically identical and topologically similar; the difference being the connectivity of elements adjacent to the surface. As with the 14 element truss both form the trailing 0.4*c* fraction of a NACA 0012 aerofoil.

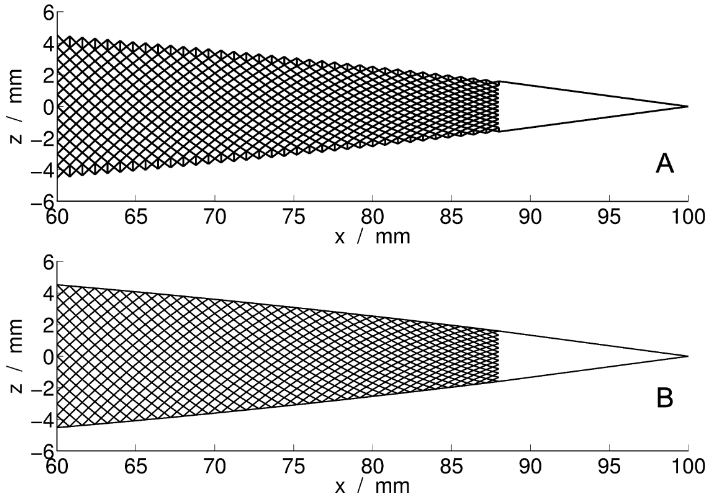

Figure 6. 1752 element truss structures with identical geometry and differing topology and referred to as structures ’A’ and ’B’.

Figure 7 gives the shape error results for optimising the location of 88 actuators within trusses A and B using the GA, SFS and IFS methods. The results for the geometry error were calculated but are not shown here. Beginning with truss A, the GA provided the best performance, and little improvement in the SSE_S value was observed at values of $\varepsilon^{max} \geq 0.1$, when using the GA search method. The GA method outperformed both the SFS and IFS procedures, however here a notable improvement of the IFS method over the SFS method was observed. This is due to the alteration of the target vector as the actuator selection process is progressed. As each actuator is added to the subset the structure is subjected to a displacement that, in the case of the shape objective, alters the target displacement. By taking large steps towards the target vector the SFS method oversteps possible improved selections that may appear as the target alters with the values of the regression coefficients. Truss B exhibited similar characteristics in the selection of optimal locations as for truss A, whereby the GA proved to be the best selection method in terms of reducing the shape error in all cases. Improved results were obtained using the IFS method when compared with the SFS method.

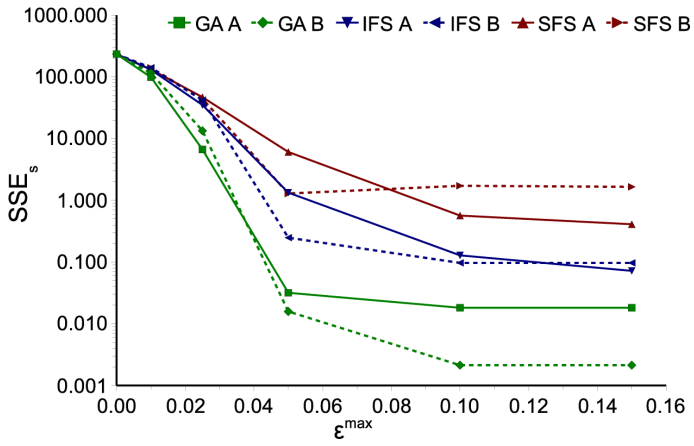

Figure 7. Shape objective (SSE_S) against maximum absolute actuation strain (ε^{max}) for trusses A and B with 88 actuator substitutions.

Comparison of the results between the two truss patterns for the GA selections revealed an improved performance of truss A at $\varepsilon^{\max} \leq 0.05$. Figure 8 illustrates the deflected structures for both A and B with 88 actuator substitutions and $\varepsilon^{\max} = 0.1$. Some common features are evident, including concentrations of actuator substitutions along the upper surfaces and between 0.6*c* and 0.75*c* on the lower surfaces of the trusses. The deflected result of truss A experienced large rotations of the elements in the upper surface region between 0.8*c* and 0.85*c* leading to a number of overlapping elements. In addition truss B required reduced strain in the upper surface element in the region 0.9*c* to *c* due to the increased deflection of the structure forward of this point.

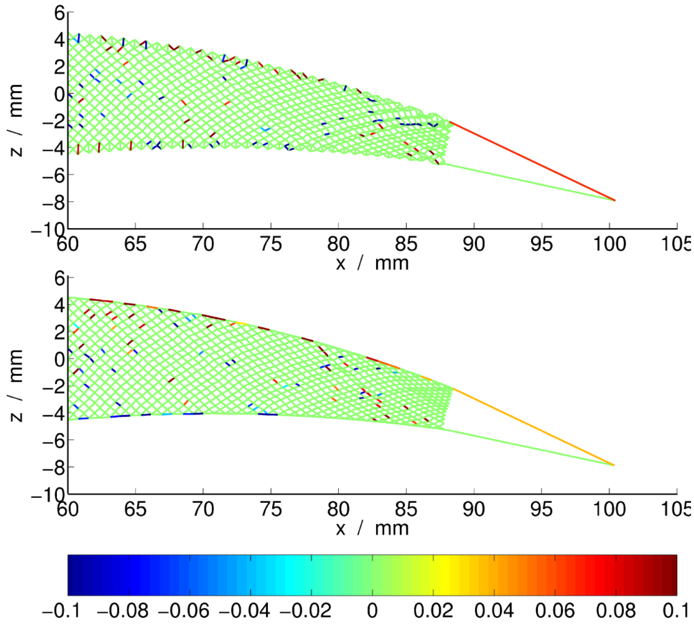

Figure 8. Trusses A and B (A above) with 88 actuators located using the GA and the shape objective. The element shade refers to the applied strain.

Consideration has thus far only been given to the effect of variation of $\varepsilon^{\max}$ on the shape errors. The number of actuator substitutions was fixed at 88 corresponding to approximately 5% of the number of elements. Suppose that the maximum actuator strain is fixed at $\varepsilon^{\max} = 0.01$ and the number of actuators are varied. Figure 9 plots the error for both shape and geometry objectives with respect to varying actuator fraction when the actuator locations were selected using the IFS method. Truss B experienced a rapid reduction in the SSE value as the actuator fraction, f_a, was increased from 0 to 0.05 and in this region provided improved performance when compared with truss A. If $f_a > 0.05$ truss A provided much improved performance with respect to truss B irrespective of the objective type and this pattern is repeated with the results using the SFS method. Figure 10 illustrates the deflected form of truss A and truss B with $f_a = 0.25$ and $\varepsilon^{\max} = 0.01$ and actuator location selections made using the IFS method. The concentration of actuator substitutions at the upper and lower surface is evident for both structures A and B due to the requirement of structural elongation and contraction of the upper and lower surfaces respectively.

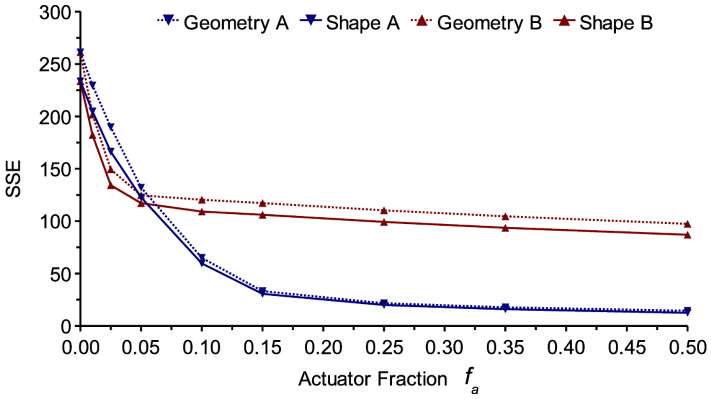

Figure 9. Sum of Squares Error (*SSE*) for both the shape objective (SSE_s) and the geometry objective (SSE_g) with respect to the actuator fraction (f_a). For all results $\varepsilon^{max} = 0.01$.

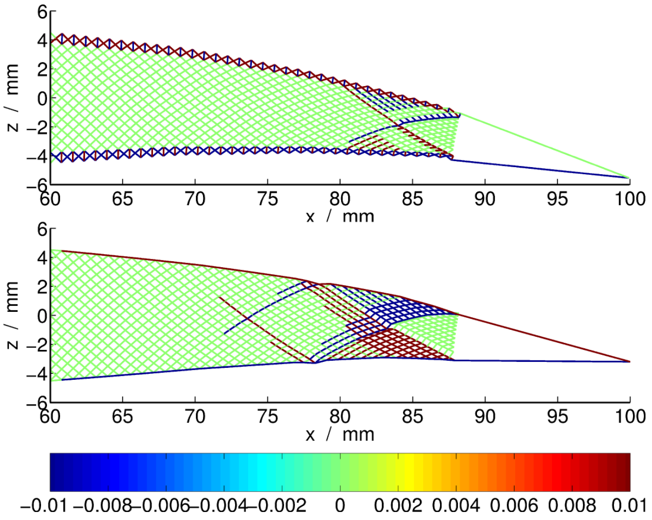

Figure 10. Deflected trusses A and B (A above) with an actuator fraction of 0.25 and $\varepsilon^{max} = 0.01$, the element shade refers to the applied strain. Of note is the increased deflection of truss A.

6. CONCLUSIONS

The use of a statically and kinematically determinate truss was proposed to act as a host for an integrated actuation system that is able to support an externally applied load. The considered trusses were 2D networks of articulated elements and in this form offer no resistance to actuation loads. However for the realisation of a practical structure the use of compliant joints between relatively stiff elements must be considered to reduce complexity. In this case the cross section of the element is varied in order to obtain localised deflection at the joint locations whist maintaining a monolithic type construction. The actuation system within the proposed structures has a structural role to fulfil and so a system is required to deliver moderate actuation strain coupled with an ability to act as a structurally integral element with minimal energy consumption. Possible smart actuation systems may include high strain piezoelectric based actuators, SMA actuators and magnetorheological actuation systems in addition to conventional hydraulic and electric actuators.

In order to find appropriate locations in which to make an actuator substitution in a previously passive element two objective functions were defined. A geometric type objective defined a set of nodal deflections based on the perturbation of the aerofoil mean line, this target deflection remained constant throughout the selection process. A shape type objective was also proposed whereby the target deflection was a function of the location of the structure at that particular phase in the selection process. This was achieved by formulating an approximation of the minimum distance between a structural joint on the surface and the target surface. This approximation offers a direct solution without the requirement of iterative root finding analysis. It was found that, except for those cases in which the SFS method was chosen, the shape objective offered improved performance.

Four methods to locate actuators within a truss using the two objective functions were investigated. An exhaustive search provided the optimum solution at the expense of high computational requirements and thus was only applicable to problems of limited size. The GA permitted the search of a fraction of the search space, however the number of function evaluations remained high and there was no guarantee of locating a global optimum. In isolated cases for the larger structures it was outperformed by IFS. IFS employs forward selection methods, however in order to reduce the problems that such methods have in large problems the regression coefficient is only incremented by a small amount at each step. In contrast SFS uses the computationally efficient method of taking greedy selection steps, either within the actuator strain limits or to a point orthogonal to the target vector. It was found that the IFS method offered significant advantages over SFS when selecting actuator locations using the shape objective, particularly when coupled to a problem in which the search space was enlarged by increased maximum actuator strain.

A simple 14 element truss was first used to provide insight in to the relative merits of the selection schemes followed by two 1752 element trusses. For examples in which the maximum actuator strain was limited to 0.01, the structure that permitted greater extension/contraction of the geometry by combining the effects of a greater number of actuators could outperform a structure in which the surface forming joints were directly connected. In most cases where computation time of a single function evaluation was sufficiently small the GA provided the best choice for actuator location selection. However it has been shown that regression methods, particularly IFS, can provide good, but sub-optimal, selection choices.

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