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Synthesis of Compliant Mechanisms for Shape Adaptation by means of a Modal Procedure

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ABSTRACT

Conventional mechanisms provide a defined mobility, which express the number of degrees of freedom of the mechanism. This allows the system to be driven by a low number of control outputs. This property is kept in case of compliant mechanisms with lumped compliance, which are obtained by replacing the conventional hinges by solid state ones. Compliant mechanisms with distributed compliance have, in general, an infinite number of degrees of freedom and therefore cannot guarantee defined kinematics. In this paper, compliant mechanisms with selective compliance are introduced. This special class of compliant mechanisms joins the advantages of distributed compliance with the easy controllability of systems with defined kinematics. This task is accomplished by introducing a new design criterion based on a modal formulation. After having implemented this design criterion in an optimization formulation for a formal optimization procedure, mechanisms are obtained in which a freely chosen deformation pattern is associated with a low deformation energy while other deformation patterns are considerably stiffer. Besides the description of the modal design criterion and the associated objective function, an application example is shown.

Keywords: Shape Control, Morphing, Compliant Mechanisms, Shape Adaptation

1. INTRODUCTION

In contrast to conventional mechanisms consisting of rigid members and classical hinges, compliant mechanisms exploit structural flexibility to produce controllable large deformations. According to a classical distinction (Ananthasuresh and Kota 1995), they can be subdivided into two groups: mechanisms with lumped compliance and mechanisms with distributed compliance. Mechanisms of the first group are obtained from their conventional counterparts by replacing classical hinges with so called solid state hinges. Usually, solid-state hinges are realised as short-length regions with reduced cross-section and therefore low bending stiffness. This involves, as a rule, poor load-carrying capability with respect to external loads. In order to avoid this problem, the second class of compliant mechanisms was developed. Mechanisms with distributed compliance make use of longer and thicker bending elements with the objective of better distributing the strain and stress over the structure.

A conventional mechanism generally behaves as a system of rigid bodies (*members*) interconnected by ideal joints. The displacement field in the mechanism is completely defined by the choice of m scalar parameters, called *mobility* of the mechanism. The m-dimensional space of possible configurations is called *kinematics* of the mechanisms.

Considering the lumped-compliance counterpart of a conventional mechanism, its motion can be analysed in a similar way as for the corresponding rigid-body mechanism: the lumped-compliance mechanism can only deform according to the configurations allowed by the system's rigid-body kinematics, regardless of the applied forces. The applied forces do not influence the system's kinematics but only the choice of a particular configuration within the mechanism's kinematics. In the following, this behaviour is described as *load-independent kinematics*. An important consequence of this behaviour is that the system can be driven by a low number of control outputs. On the other side, mechanisms with lumped compliance typically show discontinuities in their deformation pattern, which makes them not suitable for the purpose of shape adaptation, e.g. antenna reflectors (Lu and Kota 2003) or morphing wings (Campanile 2006), where a continuous deformation field is required. This requirement can be fulfilled by mechanisms with distributed compliance, which do not present sharp discontinuities. However, the kinematics of a distributed compliance mechanism is usually strongly load-dependent, which reduces their precision due to parasitic motion components. Ideally, the structure should enable smooth shape changes by keeping a high degree of stiffness in other deformation components to allow precise operation by a manageable activation system. This property identifies a third class of solid-state mechanisms, which will be denoted in the following as *compliant mechanisms with selective compliance* and which combines the advantages of both existing classes (reduced stress concentrations, a smooth deformation pattern and defined kinematics). Even if this classification and definition is new, the concept of selective compliance as such can be recognised in some standard solid-state hinges, like the cross-spring pivot or the cartwheel-hinge (Smith 2000). Due to their relative simplicity, these standard selective-compliance hinges can be easily designed by means of a simple parametric analysis. When the mechanism complexity increases, the design task becomes more challenging and must rely, as a rule, on formal optimisation techniques. Scope of the present work is to propose a general design methodology in this framework.

The design procedures for distributed-compliance mechanisms which are known from the literature mostly involve the homogenization method, the ground structure approach or the graph representation, adopted from conventional topology optimization. Based on these parameterizations, various approaches using different objective functions and constraints have been developed by several authors (e.g. Frecker et al. (1997), Sigmund (1997), Larsen et al. (1997), Saggere and Kota (1999), Kota et al. (2001), Pedersen et al. (2001) and Lu and Kota (2003)).

To sum up, the presented optimization approaches have proved their ability for designing compliant mechanisms with distributed compliance for one defined load case. If the mechanism is required to offer the above introduced selective-compliance behaviour, these approaches have to be adapted. This involves computing the structural response for many load-cases and inserting the corresponding deviations into a multi-objective cost function, to ensure a load-independent kinematics. This increases the numeric effort considerably and raises the question whether selective compliance can be enforced in a more direct way by a special optimization formulation.

A first step in this direction is represented by the modal synthesis procedure for so-called belt-rib structures, presented by Campanile (2008). Belt-rib structures are compliant wing structures which are designed to change their lift coefficient by adapting its shape and not by the use of moveable surfaces. The core of the concept, the belt rib, is a planar compliant mechanism consisting of a deformable belt connected with stiffeners (spokes) by flexible elements. The synthesis procedure arranges the spokes so that the stiffness of the desired deformation pattern (e.g. a modification of the profile camber) is nearly unchanged with respect to the belt without spokes, whereas other deformation components are strongly stiffened. However, this methodology is restricted to systems consisting of an external flexible shell and an internal stiffening structure. Besides, the procedure is applied to a special stiffener topology (each spoke is directly connected with two points of the external flexible shell without intersection with other spokes) in order to obtain an explicit condition and avoid the use of numeric optimum search algorithms.

The current work extends this synthesis procedure to design problem of a general compliant mechanisms with selective compliance.

2. DESIGN PROBLEM

Before approaching the definition of the design problem for selective-compliance mechanisms, some clarifications on the special nomenclature are necessary. While working either on the field of rigid-body mechanisms or of elastic structures, the term “degree of freedom” is unequivocal. Compliant mechanisms, however, have an infinite number of degrees of freedom (in the structure sense) and, at the same time, a finite number of degrees of freedom (mobility) in the mechanism sense. We will denote the latter with the term “kinematic degrees of freedom” in order to avoid confusion.

We consider a given design domain Ω, as depicted in Fig. **1**. The design problem for the mechanism can be seen as the problem of assigning a material to each point of the design domain (including the option of assigning a “void” material to the points in which no material is present). In the case of a mathematical representation according to continuum mechanics, this corresponds to assigning a functional relationship between the stress and the strain tensors to each point of the design domain.

The sub-domain of Ω which is occupied by material is called the *structure domain* Ω_S. Principally, Ω_S can assume an arbitrary shape within Ω that means the location of the boundaries between Ω and Ω_S are unknown except the domain on Ω_S where the kinematic boundary conditions (Ω_b) are defined and the one on which the above-mentioned load-independent kinematics is to be enforced (Ω_a). The set of degrees of freedom which are assigned to Ω_b is denoted with Γ_b and set of degrees of freedom assigned to Ω_a is called *master degrees of freedom* Γ_a. Furthermore static boundary conditions can be imposed on Γ_a. The degrees of freedom of Ω_S that do not belong to Γ_a or Γ_b is identified by Γ_c.

Even if the concept of selectable compliance does not require structural linearity, we will assume full linear behaviour in order to render the mathematical treatment simpler. Under these conditions, a set of *desired deformation modes* is defined among the design requirements. Imposing selective kinematics corresponds to requiring that the displacement field is a linear combination of the desired deformation modes. In Fig. 1 the case of one single deformation mode φ_d is shown.

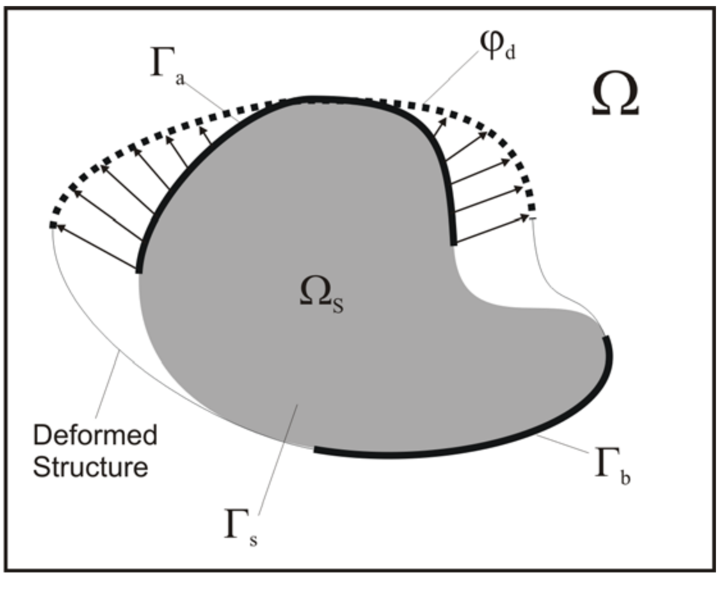


Fig. 1: Problem Specification

Apart from the load-independent kinematics, a compliant mechanism with selective compliance has to fulfil additional requirements like conciliating the defined amount of desired deformation with the range of allowable material strain (deformability). Furthermore, a certain amount of stiffness is required to carry external loads whereas on the other hand flexibility helps keeping actuation forces low (further information about the requirements to compliant structures can be found by Campanile (2006)). Concerning this work, we will primarily concentrate on the kinematics and weight of the structure. In the present case of structural linearity, the deformability requirement can easily be controlled by scaling all the beam thicknesses in order to reduce the bending stresses and the stiffness while the activation requirements can be adjusted by acting on the mechanism's width.

3. DESIGN CRITERION

A design criterion must be defined which assesses how load-independent the kinematics of a mechanical structure within the design space is. For this purpose, we consider a Finite Element model of the structure with *b* elements and *n* free degrees of freedom (i.e. degrees of freedom which not belong to Γ_b). The static equilibrium of the structure under a given external load vector $\mathbf{f} \in \mathbb{R}^n$ is described by:

$$\mathbf{ku}=\mathbf{f} \tag{1}$$

where $\mathbf{u} \in \mathbb{R}^n$ is the displacement vector and $\mathbf{k} \in \mathbb{R}^{n\times n}$ is the stiffness matrix. According to Campanile (2006) the stiffness matrix of a conventional structure is regular and well conditioned, whereas a conventional mechanism has a singular stiffness matrix, with rank deficiency equal to the mechanism's mobility *m*. For a compliant mechanism with lumped compliance (Fig. 2), the stiffness matrix is regular but badly conditioned, i.e. with a certain number of eigenvalues much smaller than the other ones. The *n* eigenvalues $\lambda_i,\ i=1\ldots n$ of the stiffness matrix are defined – together with the corresponding eigenvectors $\boldsymbol{\varphi}_i,\ i=1\ldots n$ – through the eigenvalue problem:

$$\mathbf{k}\boldsymbol{\varphi}=\lambda\boldsymbol{\varphi} \tag{2}$$

Owing to the homogeneity of (2), the eigenvectors are defined but for a scaling factor. A *normalisation* condition, e.g.

$$\boldsymbol{\varphi}^T\boldsymbol{\varphi}=1 \tag{3}$$

allows defining the elements of the eigenvector in an unambiguous way. Left multiplication of Eq. (2) on both sides with $\boldsymbol{\varphi}^T/2$ yields, by taking into account the condition (3), the energy equation

$$\frac{1}{2}\boldsymbol{\varphi}^T\mathbf{k}\boldsymbol{\varphi}=\frac{1}{2}\lambda\boldsymbol{\varphi}^T\boldsymbol{\varphi}=\frac{1}{2}\lambda \tag{4}$$

which gives insight into the physical meaning of the eigenvalues. An eigenvalue amounts to the double of the deformation energy which is required to deform the system according to the corresponding normalised eigenvector. For that reason, modes with high eigenvalues are stiffer to stimulate, that means more energy must be spent in order to reach the amplitude level defined by (3) . For a compliant mechanism with lumped compliance, the number of 'very small' eigenvalues corresponds to the mobility of the mechanism's rigid-body counterpart. We will call this number *pseudo mobility* of the lumped-compliance mechanism. Due to the low required energy, the modes corresponding to the low eigenvalues will dominate the static response of the system, i.e. the system will preferentially deform according to a displacement distribution given by a linear combination of these modes. The subspace defined by these low-energy modes will be called – analogously to the above introduced definition of kinematics of a rigid-body mechanism – kinematics of the lumped-compliance mechanism. In Fig. 2 the case of a rigid-body mechanism with one kinematic degree of freedom is shown, together with its compliant counterparts. It can be easily seen that the kinematics (i.e. the first, low energy, mode) of the rigid-body and of the lumped-compliance mechanism are very similar.

For reasons which will be clear later, we extend the eigenvalue problem (2) to the generalised problem

$$\mathbf{k}\varphi=\lambda\mathbf{m}\varphi \tag{5}$$

with $\mathbf{m}\in\mathbb{R}^{n\times n}$ as a symmetric *weighting* matrix**.** In this formulation, the relation between deformation energy and eigenvalues deviates from (4):

$$\frac{1}{2}\varphi^T\mathbf{k}\varphi=\frac{1}{2}\lambda\varphi^T\mathbf{m}\varphi \tag{6}$$

The physical meaning of the eigenvalues does not change provided that the condition

$$\varphi^T\mathbf{m}\varphi=1 \tag{7}$$

is used while normalising the eigenvectors.

If **m** is chosen as the mass matrix of the structure, this eigenvalue problem describes the vibration behaviour of the system. The *n* eigenvalues provide the square of the system's eigenfrequencies and the eigenvectors correspond to the vibration modes.

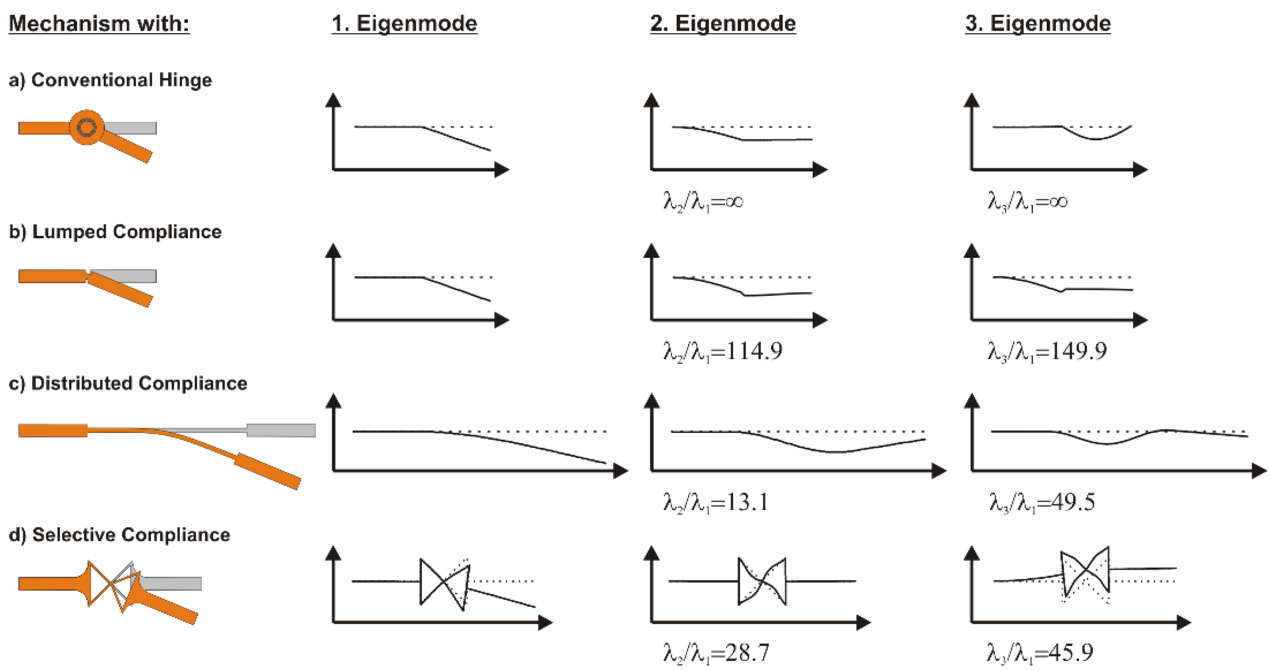


Fig. 2: Classification and eigenvalue relations of compliant mechanisms under linear assumptions (on all graphs the abscissa represents the dimensionless position and the ordinate the displacements due to deformations)

While comparing now the cases b) and c) in Fig. 2 (lumped versus distributed compliance), it can be observed that the ratio of the second eigenvalue to the first eigenvalue is much smaller in the case of mechanisms with distributed compliance. As a consequence, the mechanism is more likely to show "spurious" deformation components when loaded. The higher eigenvalue ratio in case d) shows that the cartwheel-hinge, as mentioned before, provides a lower load-dependency of the kinematics.

These observations suggest the possibility of a design criterion based on eigenvalues and eigenvectors of the stiffness matrix. A set of preferred deformation modes should be imposed as eigenvectors of the system's stiffness matrix and furthermore the ratio between the eigenvalues corresponding to the chosen eigenvectors and the remaining eigenvalues of the stiffness matrix should be as large as possible. This design task will be supported by numerical optimization and therefore the criterion must be expressed as an objective function which can be minimized by changing the system's design variables. For the sake of simplicity, we will restrict to the case of one single desired deformation mode φ_d .

4. FORMULATION OF THE OBJECTIVE FUNCTION

According to the above reported definition, only the master degrees of freedom have to fulfil the requirement of load-independent kinematics; whereas the other degrees of freedom are not subject to any restriction concerning the displacement field. It is therefore appropriate to operate on a statically condensed stiffness matrix. Accordingly, we will partition the equation system (1) as follows:

$$\begin{bmatrix} \mathbf{k}_{aa} & \mathbf{k}_{ac} \\ \mathbf{k}_{ca} & \mathbf{k}_{cc} \end{bmatrix} \begin{bmatrix} \mathbf{u}_a \\ \mathbf{u}_c \end{bmatrix} = \begin{bmatrix} \mathbf{f}_a \\ \mathbf{f}_c \end{bmatrix} \tag{8}$$

where *a* marks the *p* master degrees of freedom and *c* are the *n-p* slave degrees of freedom. The reduced stiffness matrix is obtained by applying a standard static condensation (Gasch and Knoth 1989):

$$\bar{\mathbf{k}} = (\mathbf{k}_{aa} - \mathbf{k}_{ac}\mathbf{k}_{cc}^{-1}\mathbf{k}_{ca}) \tag{9}$$

which also takes into account that no forces act on the slave degrees of freedom. Now, the system behaviour with respect to the master degrees of freedom can be described by:

$$\bar{\mathbf{k}}\mathbf{u}_a = \mathbf{f}_a \tag{10}$$

By solving the standard eigenvalue problem (2), now applied to the condensed matrix, the eigenvalues and normalised eigenvectors are computed

$$\bar{\lambda}_i, \bar{\varphi}_i, \quad i = 1 \ldots p \tag{11}$$

$$\bar{\varphi}_i^T \bar{\varphi}_i = 1 \quad \forall i = 1 \ldots p \tag{12}$$

The eigenvectors $\bar{\varphi}_i$ can be used as a basis for the description of the system's static response:

$$\mathbf{u}_a = \bar{\mathbf{\Phi}}\mathbf{a} \tag{13}$$

where $\bar{\mathbf{\Phi}}$ is the modal basis

$$\bar{\mathbf{\Phi}} = \left[\bar{\varphi}_1 \mid \bar{\varphi}_2 \mid \ldots \mid \bar{\varphi}_p \right] \tag{14}$$

The vector **a** in (13) contains the *modal coordinates*. The system equation (1) in the modal space is given by:

$$\mathbf{K}\mathbf{a} = \mathbf{q} \tag{15}$$

with the modal forces **q**:

$$\mathbf{q} = \mathbf{\Phi}^T \mathbf{f} \tag{16}$$

and the modal stiffness matrix **K**:

$$\mathbf{K} = \bar{\mathbf{\Phi}}^T \bar{\mathbf{k}} \bar{\mathbf{\Phi}} \tag{17}$$

which contains the system's eigenvalues on the main diagonal. If, by a proper structural design, one of the eigenvalues could be strongly reduced with respect to the other ones, this would allow imposing the corresponding eigenvector as the system's kinematics. If the deformation mode is to be chosen freely, an additional step in the procedure is needed which imposes the mode as one of the system's eigenvectors.

Based on the modal basis $\bar{\mathbf{\Phi}}$, a new modal basis is now obtained by replacing one mode with the desired deformation mode. In order to limit the changes in the basis to a minimum, the mode to be replaced should be as close as possible to the desired deformation mode is replaced. After computing the components of the desired deformation mode on the basis (14):

$$\mathbf{a}_d = \mathbf{\Phi}^{-1}\boldsymbol{\varphi}_d \tag{18}$$

the mode is replaced which corresponds to the largest term in the vector $\mathbf{a}_d$ .Then the new basis $\hat{\mathbf{\Phi}}$ is ordered as follows:

$$\hat{\mathbf{\Phi}} = [\boldsymbol{\varphi}_d \mid \boldsymbol{\varphi}_1 \mid \cdots \mid \boldsymbol{\varphi}_{j-1} \mid \boldsymbol{\varphi}_{j+1} \mid \cdots \mid \boldsymbol{\varphi}_p] \tag{19}$$

$$|\,\mathbf{a}_{dj}\,| = \max_{i=1\ldots p} |\,a_{di}\,| \tag{20}$$

After that, the new basis is subjected to an orthogonalization (Gram-Schmidt) procedure with respect to the stiffness matrix $\bar{\mathbf{k}}$, while keeping the first vector unchanged. After normalization, the new basis:

$$\mathbf{\Psi} = [\boldsymbol{\psi}_1 \mid \boldsymbol{\psi}_2 \mid \cdots \mid \boldsymbol{\psi}_v] \tag{21}$$

$$\boldsymbol{\psi}_i^T \boldsymbol{\psi}_i = 1, \quad i = 1\ldots M \tag{22}$$

is obtained, where $\boldsymbol{\psi}_1$ is equal to φ_d . The new basis is the solution of the new eigenvalue problem of the kind (6):

$$\bar{\mathbf{k}}\boldsymbol{\psi} = \tilde{\lambda}\tilde{\mathbf{m}}\boldsymbol{\psi} \tag{23}$$

Because $\mathbf{\Psi}$ is close to $\bar{\mathbf{\Phi}}$, it can be assumed that the weighting matrix $\tilde{\mathbf{m}}$ does not essentially differ from the unity matrix. Under this assumption, the eigenvalues can be computed as the terms on the main diagonal of the modal stiffness matrix:

$$\tilde{\mathbf{K}} = \mathbf{\Psi}^T \bar{\mathbf{k}} \mathbf{\Psi} \tag{24}$$

and give, again, a measure for the energy needed to excite the modes at the amplitude defined by the normalisation condition (22). The relevant difference with respect to the original problem with the eigenvalues and eigenvectors (11) is that one of the eigenvectors corresponds now to the freely chosen desired deformation mode. By imposing now a small ratio between the first and all other eigenvalues, selective kinematics can be imposed to the system. This criterion can be implemented into the objective function

$$f(\mathbf{x}) = \frac{\bar{\lambda}_1}{\min\left\{\bar{\lambda}_{2..n}\right\}} \tag{25}$$

where the vector $\mathbf{x}$ represents the design variables of the mechanism to be synthetised.

5. OPTIMIZATION PROCEDURE

5-1. Optimization Problem

The first step of the optimisation procedure deals with the parameterization of the structure to be designed, i.e. how the definition of Ω_S and of the material distribution on it depends from the design variables. The objective function presented in this work can be applied to any kind of structural representation like the graph-based optimization (Sauter 2008). But we will primarily focus on parameterizations in which the design variables can be seen as stiffness scaling variables (e.g. cross-sectional area, thickness or material properties) of a fixed element within the design space (e.g. the ground structure approach or the homogenization approach, which are elaborately described by Bendsoe and Sigmund (2003)). Thus, the influence of the design variables x_i on the global stiffness matrix can be described by:

$$\mathbf{k}(x)=\sum_{i=1}^{b} x_i \mathbf{k}_i \tag{26}$$

with $x_i\mathbf{k}_i$ being the contribution of the *i*-th Element to the stiffness matrix.

In addition to the elements stiffness, the chosen design variables affect the structural weight, which is our second design requirement. In order to solve the design task, the objective function has to be minimized while the weight has to be within a desired limit. In formal notation the problem can be expressed as follows:

$$\begin{aligned} &\min_{\mathbf{x}\in\mathbb{R}^b} f(\mathbf{x}) \\ &\text{subject to} \\ &W(\mathbf{x}) \le W_{\max} \\ &x_i > 0, \quad i = 1\ldots b \end{aligned} \tag{27}$$

where $f(\mathbf{x})$ is the objective function, $\mathbf{x}$ the vector of the design variables, $W(\mathbf{x})$ the weight of the structure and $W_{\max}$ the upper weight limit. Each design variable has to be larger than zero in order to avoid singularities.

5-2. Numerical implementation

An overview of the complete numerical procedure is shown in Fig. 3. By starting with an initial set of design variables the stiffness matrix of the structural configuration is obtained, which is subsequently condensed to the active degrees of freedom by means of the static condensation. After calculating the basis $\mathbf{\Psi}$, the modal stiffness matrix $\tilde{\mathbf{K}}$ is determined. This enables the evaluation of the objective function $f(\mathbf{x})$. Furthermore the value of the constraint functions, the sensitivities of the objective and of the constraint functions are determined. In order to obtain an improved design these quantities are transferred to the optimization algorithm. As an optimizer we have applied the Method of Moving Asymptotes (MMA) developed by Svanberg (1987). This algorithm allows solving smooth, non-linear constrained optimization problems with a large number of optimization variables. The method was successfully applied to different kinds of topology optimization problems and had shown its applicability and efficiency (Bendsoe and Sigmund 2003).

The above described calculation step are performed within an iterative loop. The procedure continues until convergence is reached, controlled by the change of the design variables between the iteration steps. By treating different optimization setups, we have found that the objective function is smooth but not convex. Accordingly, it may happen that the procedure converges to a local minimum. This problem can be solved by applying a multi start loop with randomly defined initial design variables.

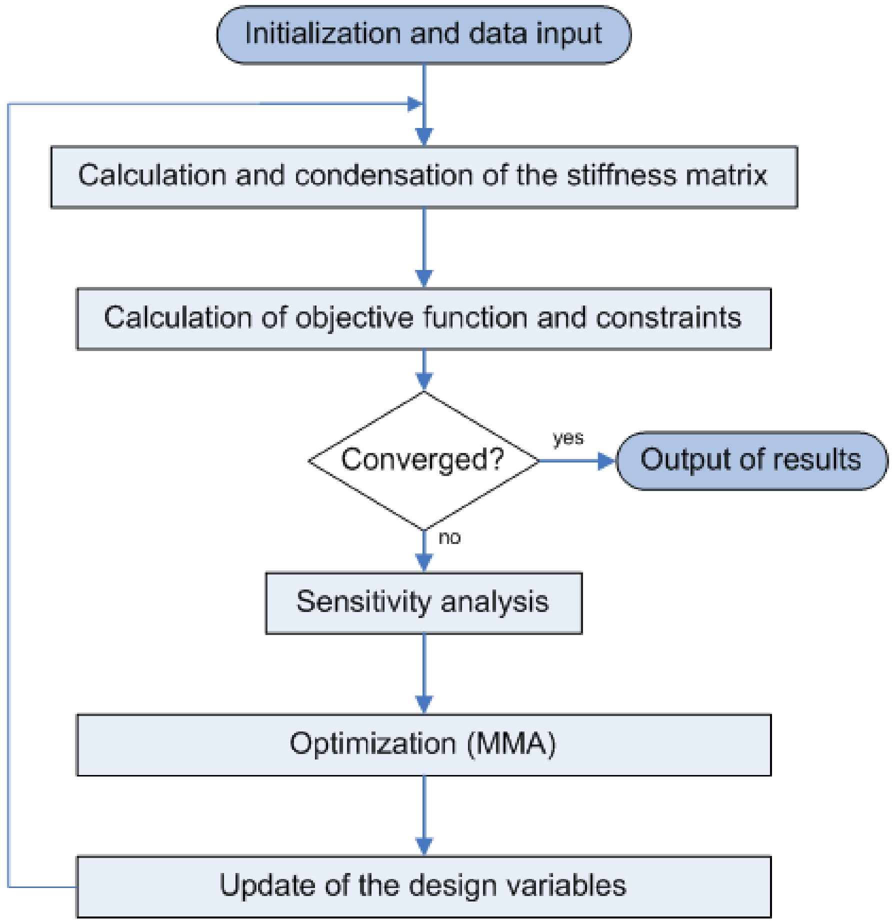


Fig. 3: Optimization process flow chart

6. DESIGN EXAMPLE AND DISCUSSION

Various optimization problems of compliant mechanisms are known from literature. Particularly demanding in terms of design are shape adaptive structures like antenna reflectors (Lu and Kota 2003), morphing wings (Campanile 2006) and adaptive car seats (Sauter 2008), where the displacements of a large set of points of the structure has to be considered. In order to show the ability of the modal objective function formulation, the current work deals with such a shape adaptive example; nevertheless, this optimization formulation can also be applied to any single-input/single-output application.

We assume a rectangular design domain as presented in Fig. 4. The bottom of the structure is fixed in all degrees of freedom (black points). On the upper surface we define master degrees of freedom (green arrows) which in the final design should show a load independent kinematics. We define a sinus shape as desired deformation mode on the vertical degrees of freedom (y-direction), and we add one single horizontal degree of freedom to avoid global motion in x-direction.

As a parameterization for the design optimization we use the ground structure approach. Thereby the problem is described as a sizing problem, where the optimizer continuously varies the cross-sectional beam areas of a fixed beam configuration. This influences the stiffness matrix and consequently the value of the objective function. The methodology described above (see Fig. 3) is implemented using MATLAB using beam elements.

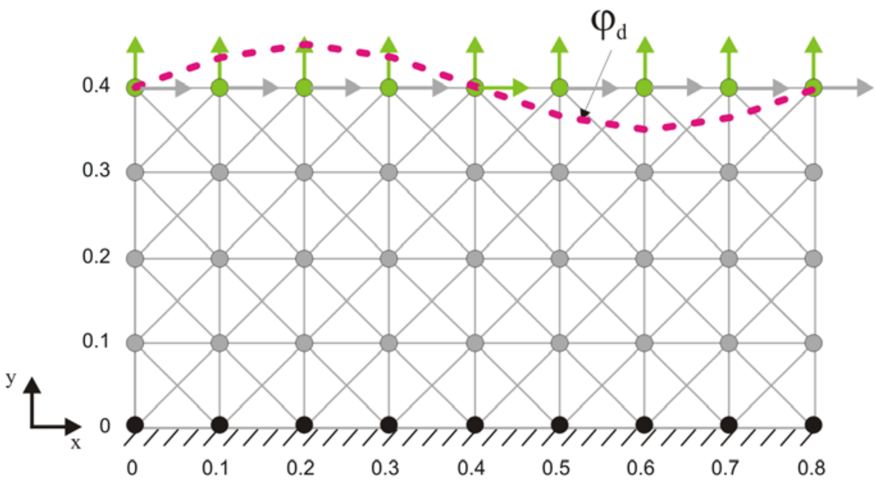
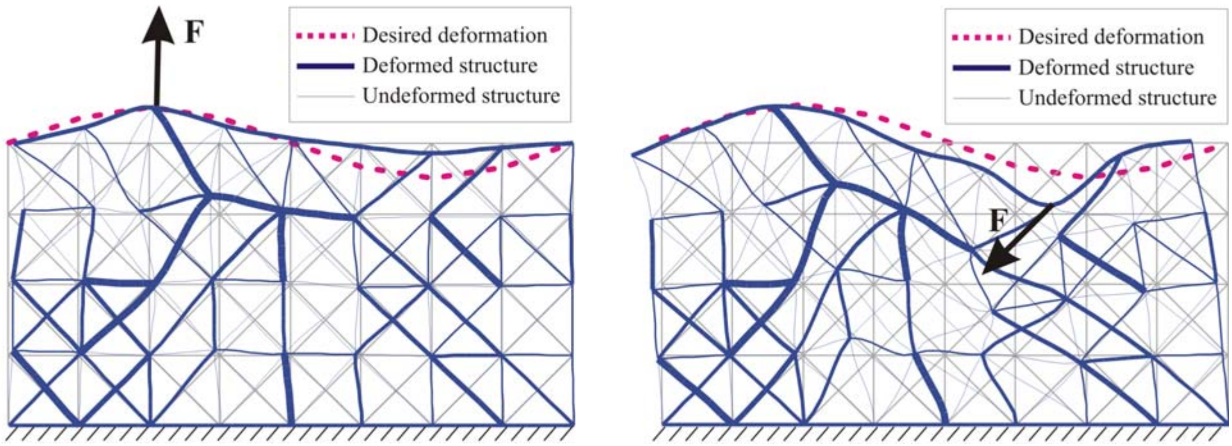


Fig. 4: Ground structure with active (green) and fixed points (black)

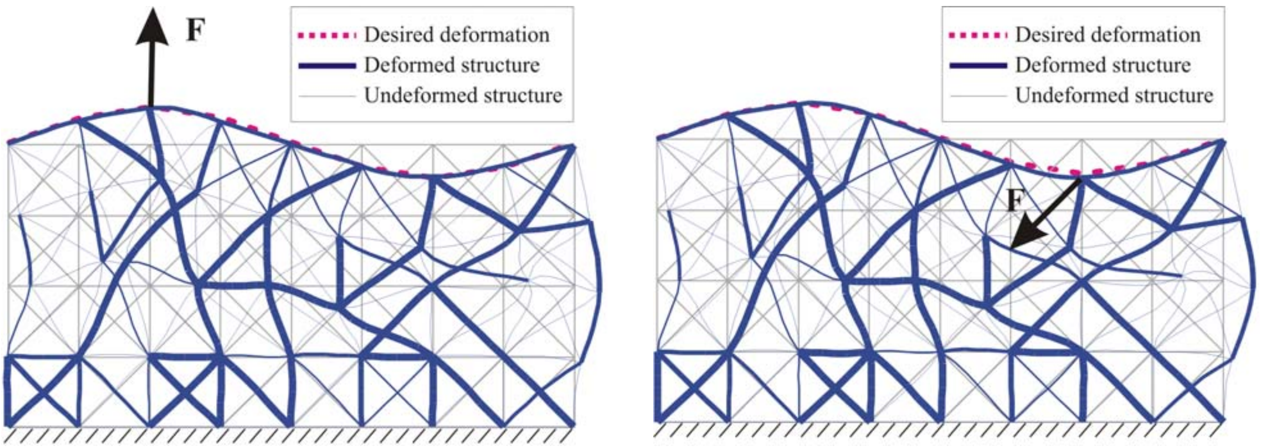
The following figures show two structural configurations with different objective function values. The deformation behaviour of the configurations under different load cases demonstrates the ability of the design procedure. Looking at the first solution (Fig. 5), in which the objective function has a value of 0.59, it can be noted that the outer shape of the deformed structure does not coincide very well with the desired mode shape (dashed red line) in both load cases. The kinematic of the compliant structure is not load independent; so the aim of designing a compliant structure with selective compliance is not satisfactory reached for that value of the objective function.



a) b)

Fig. 5 a) Load case 1 by an objective function value of 0.59 b) Load case 2 by an objective function value of 0.59

On the other hand, observing the second configuration with the objective function value of 0.04 (Fig. 6), the deformed body under both loadings fits very well with the desired mode shape.



a) b)

Fig. 6: a) Load case 1 by an objective function value of 0.04 b) Load case 2 by an objective function value of 0.04

In case of a low value of the objective function, the previous calculations have shown a defined kinematics under certain loading conditions. According to the design criterion, the desired deformation mode is expected to appear as the first eigenvector of the stiffness matrix and the corresponding eigenvalues should be small when compared to the other eigenvectors. To prove this, we have solved the eigenvalue problem of the condensed structure.

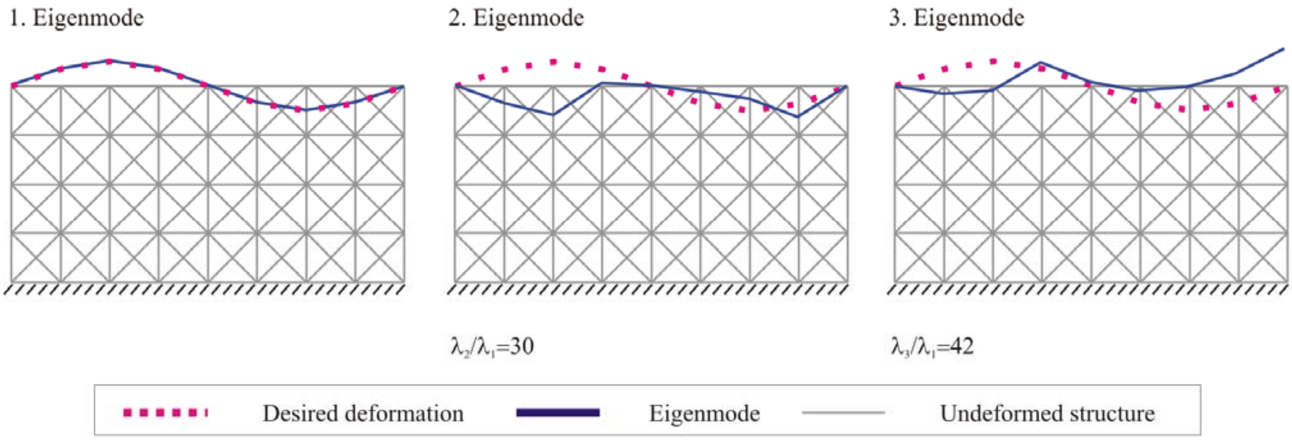


Fig. 7 Mode shapes of the optimized structure with eigenvalues ratios

Looking at Fig. 7, the desired deformation mode and first eigenvector of the structure coincide very well. Further, the ratio of the eigenvalues is comparable with the one of the cartwheel hinge presented in Chapter 2, which shows selective kinematics as desired. In the end, the results clearly point out the validity of the presented design criterion and objective function.

7. CONCLUSION

The current work introduces a new optimization problem formulation for compliant structures. A new class of compliant structures was described, which can take advantage of a distributed compliance while virtually restraining the deformation to a small number of degrees of freedom. In order to design and optimize that kind of structures a criterion was introduced and expressed as a scalar objective function by means of a modal procedure. Finally, a demonstration example (a rectangular box with a desired sinusoidal deformation mode shape at the upper surface) was calculated to show the abilities of the optimization formulation.

A conventional ground structure approach is used to represent the design space. The results underline the feasibility and the potential of the presented formulations. It was noted that the lower the value of the objective function, the more the static response of the system is dominated by the desired deformation and the less the load distribution influences the displacement distribution. We assume that the presented modal procedure leads to a smooth objective function. Another advantage of the presented formulation is the possibility of de-coupling the synthesis of the passive structure from the design of the actuator system. Since the coupling with the actuator system is realised though modal forces, a procedure is conceivable in which in a first step the structure is optimally synthesised and then the best actuator system is chosen among all variants which realise the same modal force.

In further investigations, the synthesis procedure will be extended to other constraints such as maximal allowable stress, stiffness requirements and actuation forces. In addition, nonlinearities caused by large deflections will be taken into account.

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