19th. International Conference on
Adaptative Structures and Technologies
October 6-9, 2008
Ascona, Switzerland

Vibration reduction by switch shunting of piezoelectric elements: nonlinear energy transfers between modes and optimization

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ABSTRACT

This paper deals with the reduction of structural vibrations by means of switch shunting techniques on piezoelectric elements. The main advantage of these techniques compared to resonant (inductive) shunts is that they are self-adapting on a wide band of frequencies, while requiring little energy to perform. A modal expansion of a general electromechanical system is proposed. Then, different simulations are made with 1dof. and N dof. models, in order to assess the performance and how the non-linearities of the system can affect it. These results are compared to experimental tests.

Keywords: Vibration damping, piezoelectric, shunt, switch

1 INTRODUCTION

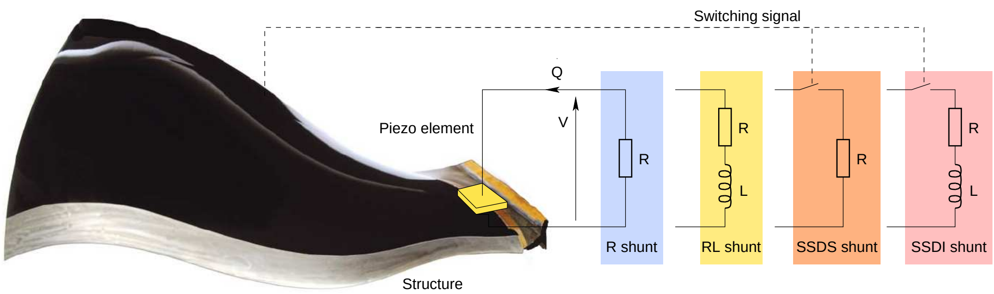

Figure 1. Structure with piezoelectric elements coupled to electric circuits to reduce the vibration.

This paper deals with the reduction of structural vibrations by means of switch techniques on piezoelectric elements. In these techniques, the piezoelectric elements are left in an open-circuit condition most of the time, with an electrical impedance briefly connected to the electrodes at specific instants [9]. This allows a temporary storage of the electric energy in the piezoelectric, which acts on the structure as a force opposed to its motion, thus reducing the vibrations. The main advantage of these techniques over resonant (RL) shunts is that they are self-adapting on a wide band of frequencies, while requiring little energy to let the current flow. This energy can even be taken from the piezoelectric element itself, making the system autonomous. Several teams [9, 3, 8, 7, 2] are working on the subject with slightly different approaches, however the basic principle remain the same. The two basic problems are the coupling of the piezoelectric element with the structure motion, and the timing of the switch.

The first part of this study is dedicated to recalling the main steps of the system electromechanical modeling, including the elastic structure, the piezoelectric patches and the electric circuit. A modal approach is used, since it is general and can be applied to any structure, provided its modal properties are known, either by an analytical approach, or by a finite-element modeling or even with an experimental modal analysis.

The second part of the study focuses on the quantification of the added damping, and the problems arising when the switch timing strategy has to deal with a large frequency band structural motion. Results from previous studies [5] show that if the structure's behavior is reduced to one mode, the efficiency depends only on the electromechanical coupling factor for this mode. However, experimental results show that the switching process, non-linear in essence, transfers energy between modes and excite high frequency modes when damping others.

The third part addresses this problem with several simulations carried out on a multi degree of freedom (dof.) system. By analyzing the results, some insights on the energy transfers are given, along with general optimization guidelines, proposed for both the piezoelectric elements and the timing of the switch.

An experimental validation of these results is proposed with an experimental setup combining different structures with an autonomous switch circuit or with a controlled switched shunt where a real-time computer controls the switch timing taking various parameters into account.

2 GENERAL ELECTROMECHANICAL FORMULATION

Previous (and incoming) studies [6, 5, 4] show that the modal expansion on the short circuit modes of the coupled electromechanical problem of a structure with a piezoelectric element is an efficient way to obtain an accurate reduced order model. It naturally introduces modal electromechanical coefficients, that are the key of the vibration reduction efficiency. This modal reduction to N modes also helps in understanding the physics of the system. Such a reduced problem writes in the following dimensionless form:

$$w(x,t) = \sum_{r=1}^{N} \Phi_r(x) q_r(t), \tag{1}$$

$$\begin{cases} \ddot{q}_r + 2\xi_r\omega_r\dot{q}_r + \omega_r^2 q_r + \omega_r k_r \displaystyle\sum_{i=1}^{N} \omega_i k_i q_i - \omega_r k_r Q = F_r, & \text{mechanical modes} \quad (2a) \\ Q - \displaystyle\sum_{r=1}^{N} k_r \omega_r q_r = V, & \text{electric circuit} \quad (2b) \end{cases}$$

where $\Phi_r(x)$, ω_r, ξ_r, k_r and F_r denote respectively the mode shape, angular frequency, the mechanical damping factor, the modal coupling coefficient and the modal forcing of the r-th mode in short circuit. The coupling coefficient k_r is both representative of the direct and indirect piezoelectric effect.

Equation (2b) can take the following forms, depending on the electric circuits shown on Fig. (1):

$$\tau_e \dot{Q} + Q - \sum_{r=1}^{N} \omega_r k_r q_r = 0 \qquad \text{R shunt} \tag{3a}$$

$$\frac{1}{\omega_e^2}\ddot{Q} + \frac{2\xi_e}{\omega_e}\dot{Q} + Q - \sum_{k=1}^{N} \omega_k k_r q_r = 0 \qquad \text{RL shunt} \tag{3b}$$

$$\dot{Q} = 0 \qquad \text{open circuit} \tag{3c}$$

In the case of a resistive shunt (R-shunt, the electric circuit is a resistance only, Eq. (3a)), τ_e is the (dimensionless) time constant of the circuit, proportional to the resistance; in the case of a resonant shunt (RL-shunt, the association with an inductance and resistance, Eq. (3b)), ω_e is the (dimensionless) angular frequency of the circuit, related to the inductance, and ξ_e is the damping factor of the circuit, related to the resistance. For the case of switch shunting, the electrical conditions are alternatively switched between open circuit (Eq. (3c)) conditions and (i) R-shunt (Eq. (3a)) for SSDS and (ii) RL-shunt (Eq. (3a)) for SSDI. The switch times are synchronized to the structure oscillations, as explained in section 3.

In [6, 5], an electromechanical model of a beam with symmetric piezoelectric elements vibrating in flexion is proposed, and the modal basis of the short-circuit modes is obtained semi-analytically. This allows the computation of all the parameters and particularly an analytical expression is obtained for k_r; in [6] this factor is shown to be the *only* parameter determining the performance of R and RL-shunts. In [5] the same factor was found to be determining the performance of SSDI and SSDS shunts. As a consequence, maximizing the modal coupling coefficients k_r is the key point to optimize vibration reduction. Different methods can be used to compute it :

- For some cases, essentially when the system geometry is simple, an analytical or semi-analytical model can be written [6];
- in [4], a finite element formulation of a structure with piezoelectric elements is proposed to compute all the modal parameters and coupling factors in a general case;
- finally, an experimental modal analysis of the system can be performed and the coupling coefficients can be approximated by the difference between the frequencies of a mode with the piezoelectric element in short circuit ω_{SC} or in open circuit ω_{OC} [1, 4]:

$$|k_r| \simeq k_{\text{eff}} = \sqrt{\frac{\omega_{oc}^2 - \omega_{sc}^2}{\omega_{sc}^2}}. \tag{4}$$

It is worth remarking however that k_r has a sign, which relates to the relationship between the direction of the motion of a mode and its influence on the piezoelectric voltage, while k_{eff}, being the ratio of electromechanical energy on potential energy is always positive [1].

3 SYNCHRONIZED SWITCH DAMPING : A 1 DOF MODEL

This section presents the results of [5]. First, the principle of Synchronized Switch Damping on Inductor (SSDI) is presented, then its performance is evaluated in simple cases where the mechanical system is reduced to 1 dof. A beam with two piezoelectric patches is used only as a test example.

The free and forced response of a system connected to a SSDI electric circuit and around the r-th resonance is investigated. Eqs. (2a) are *reduced to a one dof system* by keeping the r-th mode only. One obtains the following equations, where $u(t) \equiv q_r(t)$ denotes the only remaining modal coordinate, and the effect of the electrical charge is written like an external force in the mechanical equation's second member:

$$\text{Mechanical part:} \qquad \ddot{u} + \xi_r \omega_r \dot{u} + \hat{\omega}_r^2 u = k_r \omega_r Q + F, \tag{5a}$$

$$\text{Open circuit} \qquad \dot{Q} = 0, \tag{5b}$$

$$\text{Closed circuit} \qquad \frac{1}{\omega_e^2}\ddot{Q} + \frac{2\xi_e}{\omega_e}\dot{Q} + Q - k_r \omega_r u = 0. \tag{5c}$$

The idea behind the SSDI technique is to use the effect of the charge to reduce the beam motion. Most of the time, the switch remains open, no current flows ($\dot{Q} = 0$, Eq. (5b)) and the

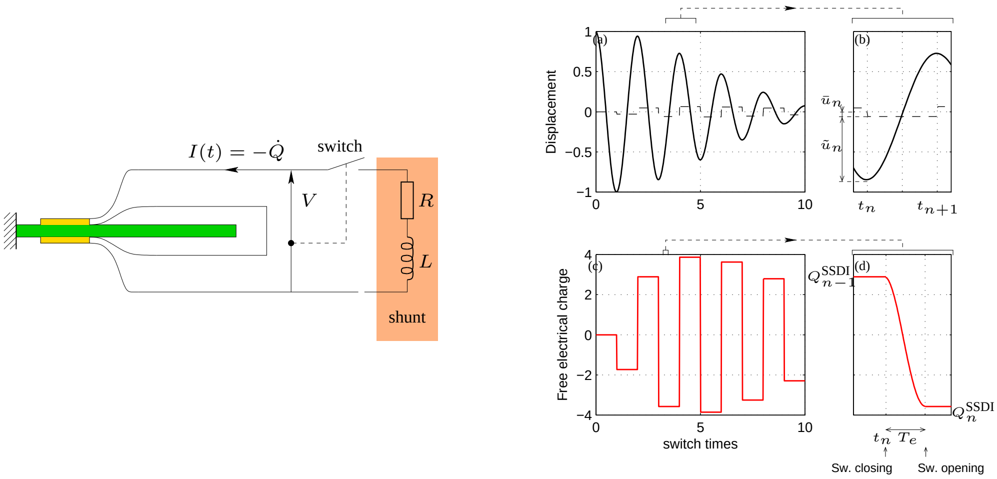

Figure 2. Beam with SSDI shunt ; time evolution of displacement $u(t)$ (a) and free electric charge $Q(t)$ (c) with zoom on one mechanical evolution step (b) and one electrical evolution step (d)

charge has an effect similar to a constant force. In order to reduce the vibrations, this force must be kept opposed to the structure's velocity. It is only when the velocity changes direction that the switch is closed and the current flows ; this instant can be found by monitoring the piezo voltage $V = Q - k_r\omega_r u$ and switching when $\dot{V} = -k_r\omega_r\dot{u} = 0$. The switch remains closed for a brief time, T_e, very small compared to the mechanical period of the structure. This time is precisely chosen as half a period of the association of the piezoelectric element and shunt, $T_e = \pi/\omega_e$. After that time, the charge should have an opposite sign and the switch is opened again. The effect of the charge is then similar to a constant force that changes of sign synchronously with the oscillations and that opposes itself to the motion, almost like a dry damper.

A different technique, called SSDS for Synchronized Switch Damping on Short, consists in the same system with a short circuit shunt. The timing of the switches and the behaviour of the system is similar, except the voltage is set to zero during the switch and the charge is lower.

3.1 Free response model

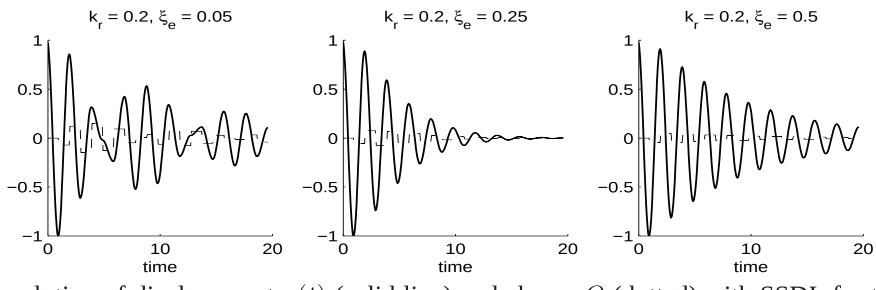

Figure 3. Time evolution of displacement $u(t)$ (solid line) and charge Q (dotted) with SSDI, for three values of ξ_e. (left): low ξ_e and beating phenomenon ; (center): optimal ξ_e improves decay rate ; (right) high ξ_e decreases decay rate.

The first model is developed for the free reponse of the system ($F_r = 0$) , without damping ($\xi_r = 0$), and with the assumption that the electrical phenomena (the changes of charge during the closed circuit phase) are infinitely quick compared to the mechanical phenomena ($\omega_e \ll \omega_r$). In that case, the time evolution of $u(t)$ can be written analitically and bounded by a decaying exponential, whose decay rate depends only on k_r and ξ_e. The case of SSDS can be treated exactly like the case of SSDI with $\xi_e = 1$.

In the case of SSDI, an optimal value for ξ_e, that maximizes the decay rate, is obtained as a function of k_r, shown on figure (4)). Then, an analytical expression of the equivalent damping

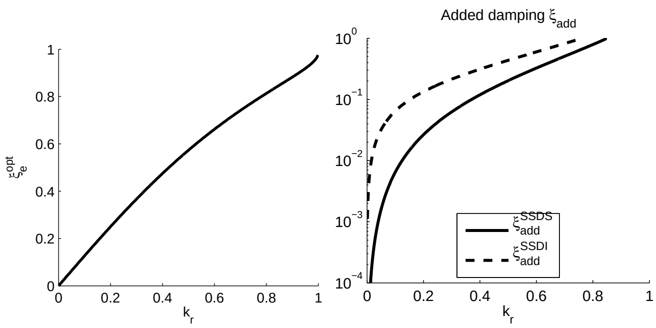

Figure 4. Optimal value of electrical damping factor (resistance) for SSDI as a function of coupling (left), and equivalent damping obtained on the structure with SSDS or SSDI (right)

factor $\xi_{\text{eq}} = \mu/\omega_r$ (defined with decay rate μ) is obtained [5]:

$$\xi_{\text{eq}}^{\text{SSDS}} = -\frac{1}{\pi}\ln\left(\frac{1-k_r^2}{1+k_r^2}\right), \quad \xi_{\text{eq}}^{\text{SSDI}} = -\frac{1}{\pi}\ln\left(\frac{1-|k_r|}{1+|k_r|}\right). \tag{6}$$

The above value of $\xi_{\text{eq}}^{\text{SSDI}}$, in the case of SSDI, is obtained for the optimal value of ξ_e. The main result is that the optimal value of ξ_e and the resulting performance depends only on k_r, exactly like in the case of linear shunts [6].

3.2 Forced response model

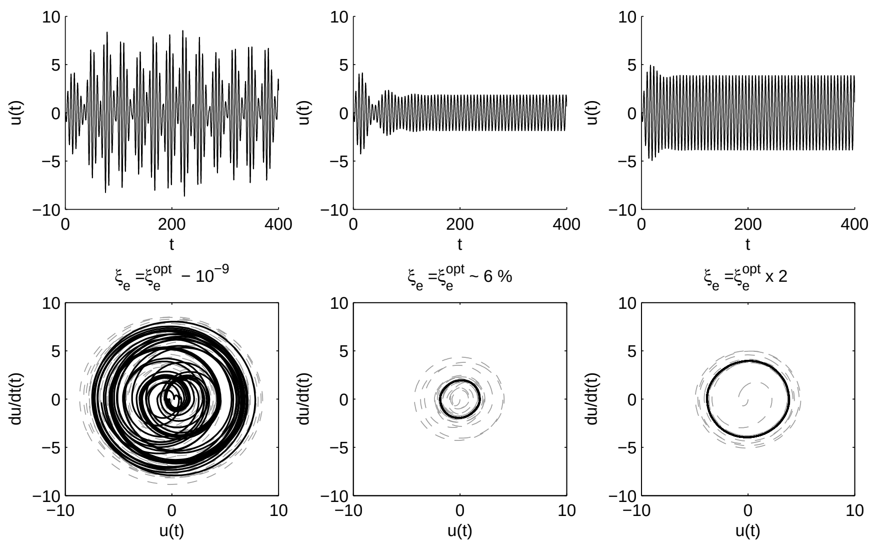

Figure 5. Time evolution of mechanical displacement $u(t)$ with SSDI and phase spaces, for $k_r = 0.2$ and $\xi_r = 0.1\%$. System forced near resonance. (left): overcritical damping ξ_e; (center) optimal damping ξ_e; (right) large damping ξ_e.

A second model is developed with the same assumption of quick electrical phenomena,, in the case of an applied external harmonic force $F_r(t) = \cos(\Omega t)$ and a mechanical damping $\xi_r \neq 0$. It allows us to compare the motion amplitude at resonance with and without switch. A semi analytical and fast way to compute $u(t)$ allows a systematic study of parameters. This study shows that a critical value of the electrical damping exists (different of the optimal ξ_e previously found) which separates the behaviour of the system between a “steady state” response and a highly irregular and inefficient timing of the switches (fig. 5).

As the reponse of the system is not harmonic, the RMS amplitude of displacement integrated over time is used as a performance indicator ; this amplitude is plotted versus the excitation frequency and the effect of the switch can be presented as an attenuation brought by the system (fig. 6,left). The optimal value of ξ_e, that maximizes this attenuation, and the resulting performance depends only on k_r and on ξ_r, exactly like in the case of linear shunts [6].

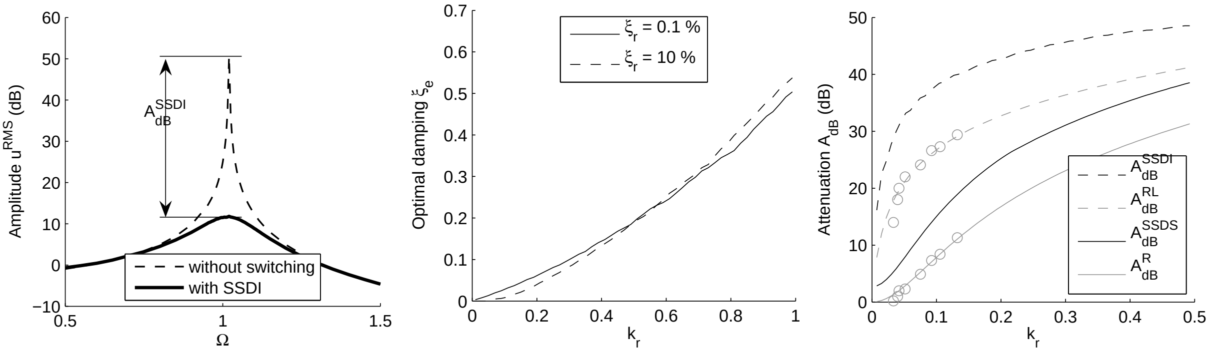

Figure 6. (left) RMS value u^{RMS} of time response $u(t)$ as a function of excitation frequency Ω, $k_r = 0.2$ and $\xi_r = 0.1\%$. (right) optimal electrical damping factor ξ_e^{opt} as a function of k_r. (center) Expected attenuation A_{dB} as a function of coupling coefficient $k_r \simeq k_{\mathrm{eff}}$ for SSDI, SSDS, and simple resistive (R) and resonant (RL) shunts, for $\xi_r = 0.17\%$. '—', '– –': theory; '○': experiments

3.3 Conclusions with 1 dof. model

The conclusions of this study are very straightforward. With the models proposed, the optimization of a SSD system can be made as follows :

- First, the coupling coefficient k_r must be maximized, by choosing the correct piezoelectric elements and placing them at selected places on the structure ;
- In the case of SSDS the performance depends only on k_r
- In the case of SSDI, the inductance related to ω_e just has to be small, in order for the electrical system to be fast compared to the mechanical part. This is a welcome difference with rhe case of linear RL shunt where L is very high and must be tuned. Then, the resistance can be chosen so that ξ_e is optimal, depending on k_r. The expected performance is very high (Fig. (6), right).

4 A N-DOF SYSTEM WITH SSDI

While experimentation with R and RL shunts showed a good agreement between 1 dof. models and experimental data, the highly non-linear behaviour of synchronized switch systems is reponsible of the excitation of various modes, and the switch timing becomes very complicated. The efficiency can be considerably lower than expected. This is why the study of N dof. systems is necessary.

4.1 Modelling

First, Eqs. (2a,3) are written in the following matrix form :

$$\begin{cases} \mathbf{M}_{oc}\ddot{\mathbf{q}}_{oc} + \mathbf{C}_{oc}\dot{\mathbf{q}}_{oc} + \mathbf{K}_{oc}\mathbf{q}_{oc} = \boldsymbol{\chi}Q + \mathbf{F}_{oc} \quad \text{and} \quad \dot{Q} = 0 & \text{open circuit} \quad (7a) \\ \mathbf{M}_{cc}\ddot{\mathbf{q}}_{cc} + \mathbf{C}_{cc}\dot{\mathbf{q}}_{cc} + \mathbf{K}_{cc}\mathbf{q}_{cc} = \mathbf{F}_{cc} & \text{closed circuit} \quad (7b) \end{cases}$$

Eq. (7a) is a N dof. system where $\mathbf{M}_{oc}$, $\mathbf{C}_{oc}$ and $\mathbf{K}_{oc}$ are respectively the mass, damping stiffness matrix in open circuit; $\mathbf{q}_{oc} = (q_1 \ \dots \ q_N)^\top$ is the vector of modal coordinates, $\boldsymbol{\chi} = (\omega_1 k_1 \ \dots \ \omega_N k_N)^\top$ is the vector of electromechanical coupling factors and $\mathbf{F}_{oc} = (F_1 \ \dots \ F_N)^\top$ is the vector of external forcing. $\mathbf{M}_{oc}$ is the $N \times N$ identity matrix, $\mathbf{C}_{oc}$ is a diagonal matrix whose i^{e} diagonal term is $2\xi_i\omega_i$ and $\mathbf{K}_{oc}$ is the diagonal short-circuit stiffness matrix with the symmetric open circuit added stiffness whose (i, j) term is $\omega_i^2\delta_{ij} + k_i k_j \omega_i \omega_j$.

Eq. (7b) is a $N+1$ dof. system that includes the electrical equation. The vector of unknowns is $\mathbf{q}_{cc} = (\mathbf{q}_{oc}^\top\, Q)^\top$, the external forcing is $\mathbf{F}_{cc} = (\mathbf{F}_{oc}^\top\, 0)^\top$ and the mass, damping and stiffness matrix in closed circuit write:

$$\mathbf{M}_{cc} = \begin{pmatrix} \mathbf{M}_{oc} & \mathbf{0} \\ \mathbf{0}^\top & 1/\omega_e^2 \end{pmatrix}; \mathbf{C}_{cc} = \begin{pmatrix} \mathbf{C}_{oc} & \mathbf{0} \\ \mathbf{0}^\top & 2\xi_e/\omega_e \end{pmatrix}; \mathbf{K}_{cc} = \begin{pmatrix} \mathbf{K}_{oc} & -\boldsymbol{\chi} \\ -\boldsymbol{\chi}^\top & 1 \end{pmatrix}. \tag{8}$$

The time evolution of the system is then computed by switching alternatively between Eq. (7a) and Eq. (7b) at specific instants introduced in the following section.

4.2 Switch timing

In the case of a one dof. model, it has been shown that switching when the mode velocity changes sign was efficient and that the voltage can be monitored to find this instant (section 3). In the case of a multimode system, one can study the mechanical and electrical power during the system evolution. The following power equations are obtained, by multipling Eq. (7a) by ${\mathbf{q}_{oc}}^\top$ and Eq. (7b) by ${\mathbf{q}_{cc}}^\top$:

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{E}_m = \frac{\mathrm{d}}{\mathrm{d}t}(\mathcal{T}_m + \mathcal{V}_m) = -\mathcal{D}_m + \mathcal{C}_m + \mathcal{P}_{\text{ext}}, \tag{9a}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{E}_e = \frac{\mathrm{d}}{\mathrm{d}t}(\mathcal{T}_e + \mathcal{V}_e) = -\mathcal{D}_e + \mathcal{C}_e, \tag{9b}$$

where

$$\mathcal{T}_m = \frac{1}{2}\dot{\mathbf{q}}_{oc}^\top \mathbf{M}_{oc}\dot{\mathbf{q}}_{oc}, \quad \mathcal{T}_e = \frac{1}{2\omega_e^2}\dot{Q}^2, \quad \mathcal{V}_m = \frac{1}{2}\mathbf{q}_{oc}^\top \mathbf{K}_{oc}\mathbf{q}_{oc}, \quad \mathcal{V}_e = \frac{1}{2}Q^2 \tag{10}$$

$$\mathcal{D}_m = \dot{\mathbf{q}}_{oc}^\top \mathbf{C}_{oc}\dot{\mathbf{q}}_{oc}, \quad \mathcal{D}_e = \frac{2\xi_e}{\omega_e}\dot{Q}^2, \quad \mathcal{C}_m = Q\dot{\mathbf{q}}_{oc}^\top\boldsymbol{\chi}, \quad \mathcal{C}_e = \dot{Q}{\mathbf{q}_{oc}}^\top\boldsymbol{\chi}, \quad \mathcal{P}_{\text{ext}} = \dot{\mathbf{q}}_{oc}^\top\mathbf{F}_{oc}. \tag{11}$$

$\mathcal{T}_m$ is the kinetic energy, $\mathcal{T}_e$ is the inductance energy, $\mathcal{V}_m$ and $\mathcal{V}_e$ are the potential mechanical and electrical (capacitance) energies; $\mathcal{E}_m$ and $\mathcal{E}_e$ are the total mechanical and electrical energies; $\mathcal{D}_m$ is the dissipated mechanical power, $\mathcal{D}_e$ is the electrical power dissipated by Joule effect in the resistance; $\mathcal{C}_m$ and $\mathcal{C}_e$ are electromechanical coupling powers and $\mathcal{P}_{\text{ext}}$ is the power of the external mechanical forces.

In Eq. (9a), the effect of the electric circuit on the mechanical part appears with term $\mathcal{C}_m$, that is the power of the electric charge during the mechanical motion $\dot{\mathbf{q}}_{oc}$. In order to reduce the vibrations, $\mathcal{C}_m$ should be kept negative. During the (predominant) open circuit phase, Q is constant and following Eq. (2b), $\dot{V} = -\dot{\mathbf{q}}_{oc}^\top\boldsymbol{\chi}$ so that $\mathcal{C}_m = -Q\dot{V}$. Finally, the product $Q\dot{V}$ should be kept positive.

By changing the sign of Q whenever $\dot{V}$ reaches zero, one expects to keep $\mathcal{C}_m < 0$. Also, instead of waiting T_e for opening the switch, one can wait the full inversion of the charge by detecting $\dot{Q} = 0$[1]. The switch condition then writes :

$$\begin{cases} \text{switch closes when} & \dot{V} = 0 \\ \text{switch opens when} & \dot{Q} = 0 \end{cases}, \tag{12}$$

as proposed in [3] for multimode control.

[1] Also, physical implementations of switches make use of diodes which limit the flow of current in one direction, effectively blocking the current when it changes direction

5 FREE RESPONSE OF A N-DOF SYSTEM WITH SSDI

The case of a free-response is studied with Eqs (7) with $\mathbf{F}_{oc} = \mathbf{0}$. Given the initial conditions, the open circuit evolution is computed, taking into account the fact that it can be obtained from Eqs (7) as a matrix product including a matrix exponential. The zero-crossing of $\dot{V}$ is numerically found and the closed circuit evolution begins then, with initial conditions obtained by continuity of the modal coordinates, velocities and electrical charge. When $\dot{Q} = 0$, the open circuit evolution begins again.

5.1 Influence of neighboring modes

A simulation is made with two modes of dimensionless angular frequencies $\omega_1 = 1$ and $\omega_2 = 1.3$, no mechanical damping and coupling coefficients $k_1 = k_2 = 0.1$. The electrical shunt used has a high angular frequency $\omega_e = 20$ and an electrical damping of 0.1, theorically optimal from the 1 dof model (Fig. 4). The initial condition is a non-zero displacement of mode 1 only ; the results are plotted on figure 7. The total energy is $\mathcal{E} = \mathcal{E}_m + \mathcal{E}_e$, as defined by Eq. (9a). Mode 1 and 2 modal potential energies are $\mathcal{V}_1$ and $\mathcal{V}_2$, defined by $\mathcal{V}_i = 1/2\,\hat{\omega}_i^2 q_i^2$, with $\hat{\omega}_i^2 = \omega_i^2(1 + k_i^2)$. One has $\mathcal{V}_m \simeq \sum \mathcal{V}_i$, by noting that the stiffness matrix is almost diagonal.

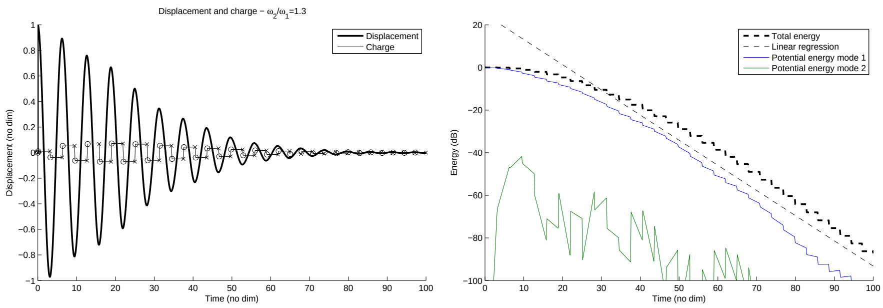

Figure 7. Evolution of displacement, charge and energies

The second mode is excited by the switching process, even with the initial condition only on mode 1. However, the energy is quickly decaying, with an exponential decay ; this decay can be used to compute an equivalent added damping :

$$\mathcal{E} \simeq \mathcal{E}_0 e^{-2\mu t}; \qquad \mu = \omega_1 \xi_{\text{eq}}$$

A second simulation is made with $\omega_2 = 3$ and the results are shown on figure 8. This time, the second mode is much more excited, and the switching is then perturbed ; it occurs too frequently, there is no charge buildup in the circuit. The energy decay rate is lower.

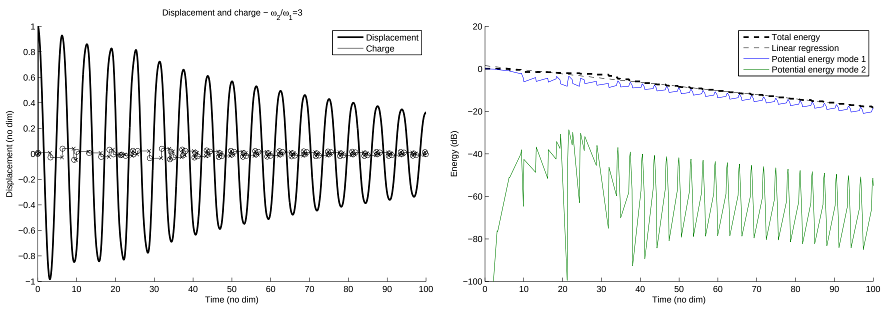

Figure 8. Evolution of displacement, charge and energies

In order to study the influence of neighboring modes, the same simulation is repeated with several ratios of the modes angular frequencies ω_2/ω_1. For each simulation, an equivalent damping is deduced from the decay rate of the total energy and the result is plotted on figure 9.

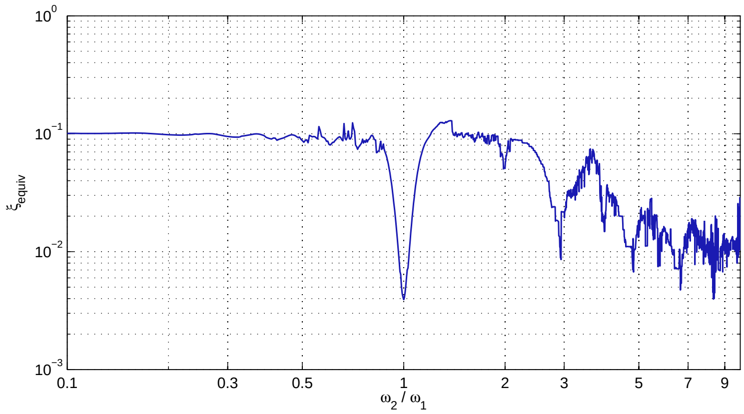

Figure 9. Equivalent damping of the first mode as a function of the ratio of frequencies ω_2/ω_1

The following conclusions can be made :

- The damping is not affected by lower frequency modes, and is almost identical to the 1 dof model prediction ;
- the damping is low when the two modes are close to each other in frequency ($\omega_2 \simeq \omega_1$). What happens is that they begin to oscillate in phase opposition, a motion which is not well coupled with the electric circuit ;
- the efficiency is lower when the upper mode has a frequency which is a multiple of the damped mode ; this is especially true for odd multiples. This is due to the fact that the upper mode is excited by the repetitive shock effect of the charge, a signal with a roughly square shape, mainly composed of odd harmonic components (Fig. 2). Mode 2 motion disturbs the damping of the first mode by desynchronizing the switch with the first mode ;
- finally, high frequency modes have an impact on the performance.

5.2 Influence of inductance

As the excitation of high frequency modes can decrease efficiency (and experimentally generate annoying noises), it may be interesting to use a higher inductance, increasing T_e and "softening" the shocks in the structure. First, the efficiency on a 1 dof. system[2] with various inductances is tested and the equivalent damping plotted versus ω_e on Fig. 10. It shows that the performance is correct for $\omega_e > 3\omega_1$. Below that value, the switch is too slow to accumulate enough charge.

Then the same simulation is made, with a 2 dof model with second mode at $\omega_2 = 7.5$. The results (fig. 10) show that :

- for $3 < \omega_e < \omega_2$ the performance is satisfactory, mode 2 is only slightly disturbed ;
- when $\omega_e \simeq \omega_2$ mode 2 is excited and the performance is very low ;
- when $\omega_e > \omega_2$ mode 2 is less excited, but still affects performance.

One can remark that one of the main goals of switch techniques is to use a passive inductor instead of a synthetic (semi-passive) inductor required for big values of T_e ; hence high inductances are not really an alternative.

[2]This computation requires a 1+1 dof. model to take the electrical period into account

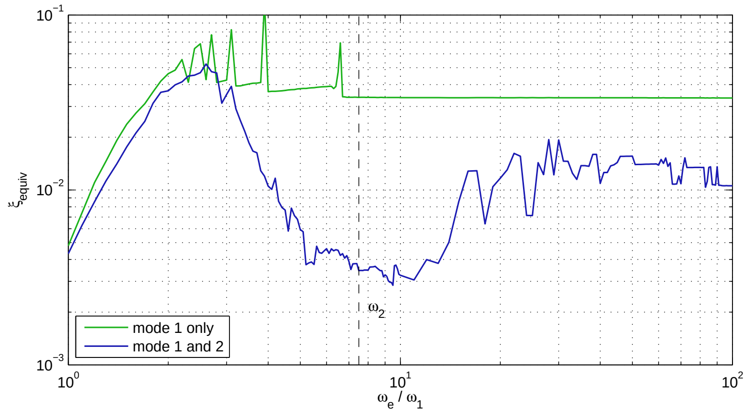

Figure 10. Equivalent damping as a function of the ratio of frequencies ω_e/ω_1, without and with second mode.

6 FORCED RESPONSE OF A N-DOF SYSTEM WITH SSDI

We now analyze the response of the system when an external force is applied. The system evolution is simulated with a Newmark algorithm to integrate Eqs. (7) with $\mathcal{F}_{oc} \neq 0$.

To avoid the detrimental high frequency switching (like in the case of Fig. 8), the switch closing condition is now:

$$\text{circuit closes when} \quad \dot{V}_f = 0 \tag{13}$$

where $\dot{V}_f$ denotes an alternative derivated voltage signal, which can be:

- $V(t)$ filtered by a lowpass filter, which is representative of the synchronisation method found in certain switching systems [9];
- $V(t)$ filtered by a modal filter, which allows the user to target specific modes and weight their influence on the vibration reduction [3];
- the voltage of another piezoelectric element used as a sensor only and designed to couple specifically to targeted modes [8].

6.1 Experimental setup

The experimental setup is built around a cantilevered beam with piezoelectric elements. The beam is excited near the tip by a glued magnet, in the field of a coil, which exerts a force proportional to the current. The velocity is measured at the driving point by a laser velocimeter. Both contactless devices avoid electrical grounding of the beam. Two circuits have been used: (i) an autonomous SSDI shunt provided by LGEF Lyon[9] or (ii) a switch with exchangeable shunts controlled by a real-time computer running Mathworks xPC target, in order to implement quickly various filters and switching strategies, based on piezoelectric voltage or other sensor data. The data acquisition can be in time domain or in frequency domain with averaging. A sweep test performed on a beam with optimized coupling (high k_1, k_2 and k_3) shows a strong cross excitation of the second mode at the first resonance (fig. 11). Those experimental results are compared to a simulation, showing good agreement.

6.2 Selective coupling

Experimentation reveals (fig. 12) that when the beam is excited at first resonance, the switching excites the second mode and is partially desynchronized. In order to test the influence of that

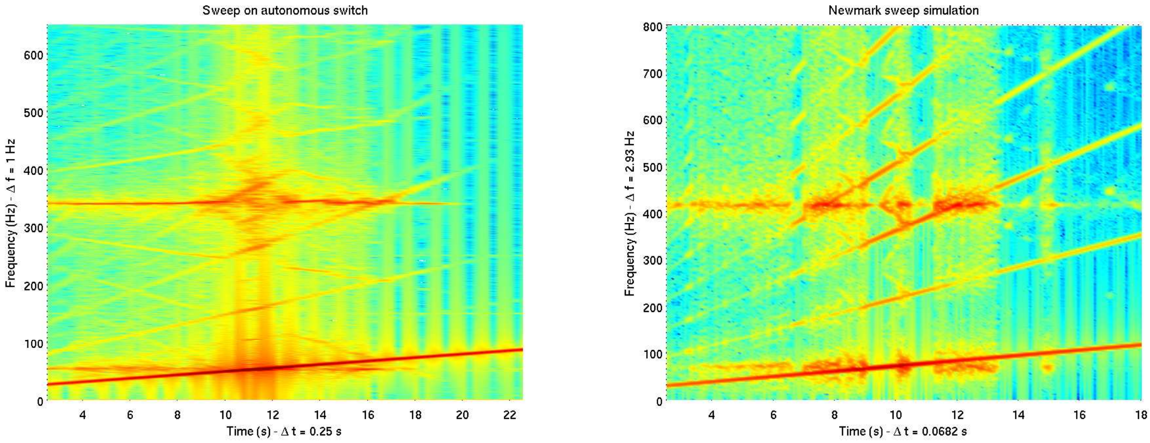

Figure 11. Time/frequency diagram of beam tip velocity with sweep around first resonance

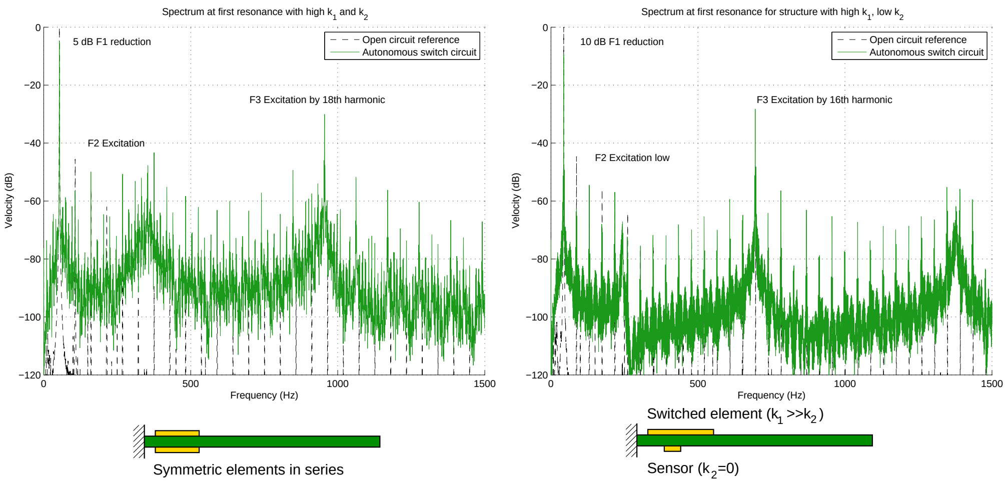

Figure 12. Spectrums for excitation at resonance of two beams with different coupling optimization

phenomenon, a second beam is built with an optimized high k_1 and low k_2. Aditionally a piezoelectric sensor for controlling the switch timing is available, with $k_2 \simeq 0$. The results show a better performance, especially in terms of stable synchronization of the switch.

6.3 Influence of inductance

The use of a high passive inductance to completely avoid the excitation of upper modes is impractical for most of the piezoelectric elements (which have a low capacity). However, the choice of the inductor allows experimentally (fig. 13) to reduce the excitation of high frequency modes (> 5 kHz), responsible of the annoying noise emission when a low frequency mode is attenuated.

7 CONCLUSION

Several simulations were carried out on SSDI systems, with different assumptions. Simple models allow to draw general conclusions : the performance depends on the modal coupling coefficient k_r for a given mode. However, the complexity of the switch timing problem arises as soon as a N dof. model is used. The efficiency quickly degrades but measures can be taken :

- by optimizing the piezoelectric element, one can avoid coupling with the modes which are not of interest and degrade the performance ;

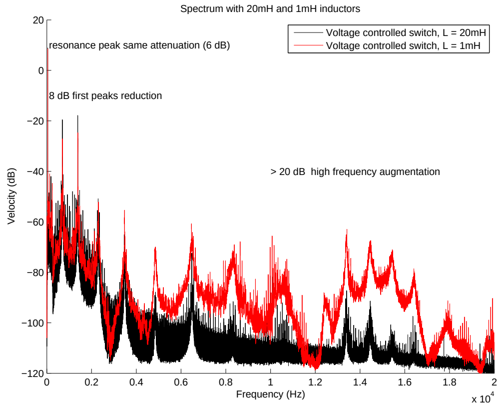

Figure 13. Spectrums for excitation at resonance with different inductors

- by choosing the inductance, one can avoid the high frequency swithing noise, and maybe in some cases reduce some cross-excitation of modes ;
- finally, the most important and difficult problem in switch shunting is the timing of the switch. The careful filtering of the piezoelectric voltage allows good experimental results, but complex filters are difficult to passively implement. Preliminary research indicates that some simple criteria based on voltage and charge in the piezoelectric element may help to optimize the vibration reduction ; however, the physical implementation may prove difficult.

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