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Flutter Boundary Prediction of Smart Adaptive Aeroelastic Systems

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ABSTRACT

This paper presents a preliminary study on flutter boundary prediction of a smart *adaptive* aeroelastic multimode system, in which flutter takes place due to a coupling between higher aeroelastic modes during the process of structural adaptation, while the flow speed is kept fixed. In such a new situation the conventional damping method would hardly work well. The prediction approach based on Jury's stability criterion in the discrete-time process will be applied. Numerical results show that Jury's criterion is much more effective in predicting the flutter boundary than the damping coefficient of the flutter mode.

Keywords: Flutter boundary prediction, adaptive aeroelastic system, Jury's stability criterion

1. INTRODUCTION

Flutter test is always carried out with great attention, because an explosive flutter often sets in unexpectedly with a small increase in speed. For an ordinary airplane, the flight speed (or the dynamic pressure) is the only variable to the instability, provided that Mach number is fixed. The higher the flight speed is, the narrower the margin for stability is. This is also true for a morphing airplane with adaptive wings if it flies while keeping structural properties of the wings unchanged. However, during the process of structural adaptation of the wings, their structure and a system for adaptation would become movable and less rigid, so that the adaptation might lead to *coalescence* of the frequencies of aeroelastic modes of the wings, even though the flight speed is fixed. This presents a newly emerging situation for flutter.

Flutter boundary prediction methods in which the information is extracted solely from measured response signals of the wings are classified into two categories: 1) evaluation of the damping of the flutter mode, and 2) estimation of stability of the aeroelastic wing system by using (a) Flutter Margin based on the Routh stability test in the continuous-time domain [1, 2] or (b) Jury's stability criterion or flutter margin for discrete-time systems founded on Jury's criterion in the discrete-time domain [3-14].

Although the damping approach is traditionally very popular to aeroelasticians, there are often essential difficulties. Analytically the damping coefficient of the critical flutter mode shows little variation until a drastic and sudden drop occurs near the boundary. Furthermore, damping coefficients measured in actual tests were very scattered particularly in a range close to the boundary [15, 16].

To examine directly the stability of the wing system, Zimmerman and Weissenburger [1] proposed a parameter called Flutter Margin, applicable only to a binary-mode flutter system. This parameter is based on the Routh stability test of the system and expressed in terms of the frequencies and dampings of the two modes coupled to cause flutter. Analytically, Flutter Margin shows a monotonic decrease in its value over a wider speed range. As mentioned above, however, an accurate evaluation of the true dampings is usually very difficult. Damped oscillations are easily affected by air turbulence. Hence, two stochastic means, i.e., the histogram and the probability density function were recently incorporated in Flutter Margin to account its uncertainty [2].

Jury's stability criterion [17] is defined by the coefficients of the characteristic equation of the system expressed in the discrete-time process, just like Routh's stability test by those of the continuous-time representation. The first application of Jury's criterion with the use of an ARMA modeling [18] to response signals of a wing model test revealed its superiority over the conventional damping method in predicting the flutter boundary [3]. Jury's criterion was also reported to be effectively applicable to the estimation of divergence boundary of forward-swept wings [7, 19]. More recently, a new stability parameter, named "flutter margin for discrete-time systems (FMDS)" was proposed by combining two parameters of Jury’s criterion [5, 6]. FMDS showed remarkably monotonous behaviors characterized by an almost linear relationship with respect to the dynamic pressure of the flow. The superiority of FMDS was very recently demonstrated over two other approaches in numerical examples [11]. Most studies mentioned above were mainly associated with flutter analyses or tests conducted on stationary conditions. It should be noted, however, that the Jury criterion and FMDS were also applied to nonstationary flutter analyses and tests in which the dynamic pressure increased at a constant rate [7-9]. FMDS was also successfully applied to flutter of a two-dimensional adaptive morphing wing [12-14] which would occur in a "nonstationary" process of adaptation represented by a gradual change in the natural frequency of the heaving or the pitching mode while the flight speed was fixed. A brief review [20] on flutter prediction based on the Jury criterion and FMDS was presented from a viewpoint of an adaptive wing.

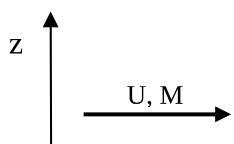
All studies based on Jury’s criterion or FMDS were concerned with the binary-mode flutter, except for Ref. 10, in which an extended FMDS was proposed in an ad hoc manner to apply to a cantilever wing with a flap. As pointed out in [11, 20], the extended FMDS for the binary-mode case is not equivalent to the original FMDS. In addition, it was reported that the extended FMDS did not show a monotonic behavior for a low-aspect-ratio wing [11].

It is, therefore, necessary to exploit a reliable flutter prediction of a multimode system based on Jury's criterion. For this purpose, a preliminary analysis on supersonic flutter of a two-dimensional multimode panel supported with a distributed spring was presented [21]. Panel flutter has well been understood so that one may easily construct an aeroelastic system in which a higher mode is involved in the occurrence of flutter. In addition, its stability characteristics are essentially similar to those of airplane wings. In the present study, we will extend the multimode panel system used in the previous analysis [21] to an *adaptive* one in which the stiffness of each mode can be changed continuously. Using this system, we may examine the usefulness of Jury's stability criterion for estimating the instability caused by the coalescence of the frequencies of two aeroelastic higher modes due to a mal-adaptation of the panel stiffness.

2. ANALYSIS

A two-dimensional panel is exposed to a supersonic flow streaming along the x-axis, as shown in Figure 1. The panel is simply supported at the leading and trailing edges, i.e., $x = 0$ and l. The z-axis is taken normal to the x-axis and the motion of the panel is given by $w(x, t)$, where t is time. The aerodynamic force, $p_a(x, t)$, acts on the upper surface of the panel, which is also supported by a distributed spring with spring constant k, and its restoring force is given by $kw(x, t)$.

The equation of motion of the panel is given [22] by



$$D\frac{\partial^4 w}{\partial x^4} + p_a + kw + \rho h \frac{\partial^2 w}{\partial t^2} = 0\,, \tag{1}$$

where D, h, and ρ are, respectively, the stiffness, thickness and mass density of the panel. The aerodynamic force is assumed in the form of piston theory [22] as

$$p_a = \frac{\rho_a U^2}{M}\left(\frac{\partial w}{\partial x} + \frac{1}{U}\frac{\partial w}{\partial t}\right), \tag{2}$$

where M, U and ρ_a are the Mach number, air velocity and density of the supersonic free flow, respectively. The deflection of the panel is assumed by using the first four modes:

$$w(x,\ t) = \sum_{m=1}^{4} w_m(t)\sin\frac{m\pi x}{l}\,, \tag{3}$$

where $w_m(t)$ is the amplitude of the m-th mode. Applying the Galerkin method to Eq. (1) and introducing a vector defined by

$$\boldsymbol{w} = \{w_1\ w_2\ w_3\ w_4\ v_1\ v_2\ v_3\ v_4\}^T \text{ with } w'_i = v_i \ \text{ for } i = 1,\ \ldots,\ 4, \tag{8}$$

we obtain the state differential equation as

$$\boldsymbol{w}' = \boldsymbol{A}\boldsymbol{w}\,, \tag{9}$$

where a prime represents the differentiation with respect to time, and

$$\boldsymbol{A} = \begin{bmatrix} AA & AB \\ AC & AD \end{bmatrix},\ AA = \begin{bmatrix} 0&0&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \\ 0&0&0&0 \end{bmatrix}, AB = \begin{bmatrix} 1&0&0&0 \\ 0&1&0&0 \\ 0&0&1&0 \\ 0&0&0&1 \end{bmatrix},$$

$$AC=-\begin{bmatrix} d_1+K_1 & r\varphi(1,2) & r\varphi(1,3) & r\varphi(1,4) \\ r\varphi(2,1) & d_2+K_2 & r\varphi(2,3) & r\varphi(2,4) \\ r\varphi(3,1) & r\varphi(3,2) & d_3+K_3 & r\varphi(3,4) \\ r\varphi(4,1) & r\varphi(4,2) & \varphi(4,3) & d_4+K_4 \end{bmatrix}, AD=-\begin{bmatrix} \gamma & 0 & 0 & 0 \\ 0 & \gamma & 0 & 0 \\ 0 & 0 & \gamma & 0 \\ 0 & 0 & 0 & \gamma \end{bmatrix} \quad \text{with } K_i=\kappa. \tag{10}$$

$$\gamma=\frac{\rho_a a_a}{\rho h},\ d_i=i^4\omega_1{}^2,\ \kappa=\frac{k}{\rho h},\ r=\frac{2\gamma}{\ell},\ \omega_1=\left(\frac{\pi}{l}\right)^2\sqrt{\frac{D}{\rho h}},$$

$$\varphi(i,\ m)=\frac{im\left\{1-(-1)^{i+m}\right\}}{i^2-m^2} \text{ for } m\neq i \text{ and } \varphi(i,\ i)=0.$$

From now on, we assume that the values of K_i (i=1, ..., 4) can independently be assigned, although they are originally equal to each other as shown above by κ [21]. From Eq. (9), we obtain the characteristic equation in the continuous-time system:

$$\prod_{i=1}^{4}\left(s-s_i\right)\left(s-s_i^*\right)=0, \tag{11}$$

where an asterisk over a variable denotes a complex conjugate of the variable. Taking the z transformation of equation (11), we have

$$F(z)=\prod_{i=1}^{4}\left(z-z_i\right)\left(z-z_i^*\right)=0, \tag{12}$$

where z_i and z_i^* correspond to s_i and s_i^*, respectively, and z_i is defined by

$$z_i=\exp(s_iT), \tag{13}$$

where T is a sampling period. Equation (12) is rewritten as

$$F(z)=\sum_{i=0}^{8}a(i+1)z^i \quad \text{with } a(9)=1. \tag{14}$$

When the system under consideration is stable, Jury's stability criterion is expressed by

$$F(1)>0,\ F(-1)>0 \text{ and } \left|X_{i-1}\pm Y_{i-1}\right|>0 \quad for\ i=2,\ 4,\ 6,\ 8. \tag{15}$$

As for the four-mode system, the dynamic stability is usually determined by X_7 -Y_7. The submatrixes X_{i-1} and Y_{i-1} are given in Appendix.

3. NUMERICAL EXAMPLE AND DISCUSSION

Like in [21], again we have assumed a *smart* distributed spring which may stiffen each natural vibration mode of the panel independently. Furthermore, here we introduce a hypothetical *adaptive* system in which the stiffness, i.e., the natural frequency of a certain selected mode is variable without affecting the remaining modes. Using this adaptive system where coalescence of the two natural frequencies is possible, we will compare the feasibility of the present approach based on Jury's stability criterion with the damping method in predicting its flutter boundary. Numerical values to be used in the calculation are:

l=100 [mm], h=0.1 [mm], E=1.28x10^4 [kg/mm^2], ρ_m =8.13x10^{-10} [kg sec^2/mm^4],

ρ_a =1.0 x10^{-5} [kg sec^2/mm^4], γ =3.0 [1/sec], T =0.007 [sec]

Without any support from the spring system, the natural frequencies of the four modes, f_1 to f_4, were 18.0, 72.0, 161.9, and 287.9 [Hz], and the panel flutter was predicted to set in at the dynamic pressure q=9.17x10^{-5} [kg/mm^2] due to a coupling between the first and the second mode. To decouple the second mode from the first, we will make only K_2 increase from zero while K_1, K_3, and K_4.remain zero.

In Figure 2, the natural frequencies of the four modes calculated at q=0 are plotted against K_2. With increasing K_2 the natural frequency of the 2nd mode approaches that of the 3rd mode. A series

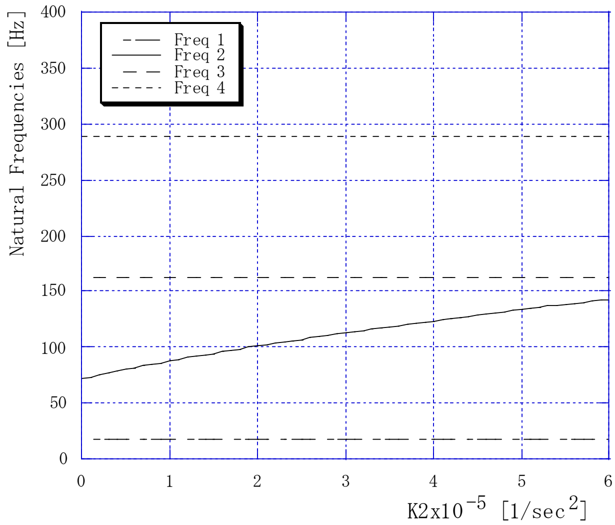


Figure 2. Natural frequencies versus K_2.

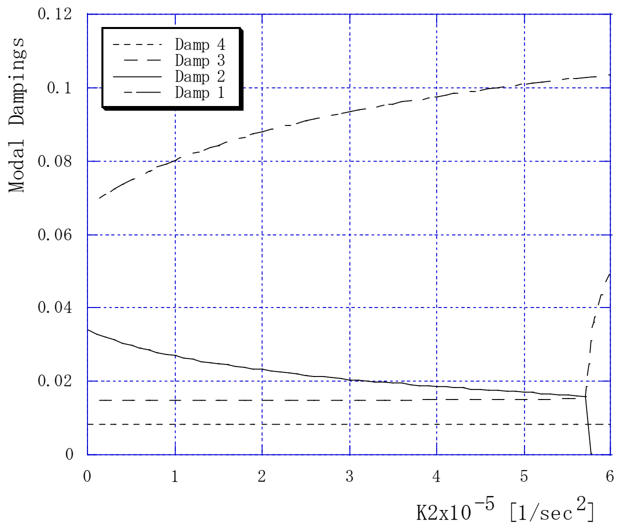


Figure 3. Modal dampings versus K_2.

of the flutter analysis was carried out by changing K_2 with the dynamic pressure being fixed at q=4.0x10^{-5} [kg/mm^2]. Figure 3 illustrates the dampings of the four modes against K_2. As K_2 increased, the damping of the first mode increased whereas that of the second mode decreased gradually and suddenly changed its trend around K_2=about 5.7x10^5 [1/sec^2] with a sharp drop to zero at K_2=5.8x10^5 [1/sec^2]. Reacting to the second mode, the 3rd mode damping increased rapidly. The flutter boundary with respect to K_2, $(K_2)_f$, was 5.8x10^5 [1/sec^2], where the frequencies of the second and the third mode coalesced.

The parameter *X7-Y7* is plotted against K_2 in Figure 4, where its value decreases very quickly around K_2=3.5x10^{-5} [1/sec^2] and then it is reduced rather uniformly between K_2=4.0x10^{-5} and 5.0x10^{-5} [kg/mm^2] and continues to remain very small but positive up to K_2= 5.8x10^{-5} [1/sec^2]. In this analysis no structural damping was taken into account. Hence, we will skip a further discussion of stability in the range above K_2= 5.0x10^{-5} [1/sec^2], because the inclusion of the damping would change the aspect of stability somehow different in the third phase. It is necessary to study this point further.

From Figure 4 it is clear that one may observe easily the approaching to the boundary through the three phases; the first rapid drop of *X7-Y7* around K_2=3.5x10^{-5} [1/sec^2], the following steady decrease in *X7-Y7* and the final stage with very low stability. On the other hand, in Figure 3 the damping of the critical mode remained sufficiently high up to about K_2=5.7x10^5 [1/sec^2], and quite suddenly dropped very quickly to zero. It is very difficult to predict the appearance of instability in advance.

In the previous flutter analysis [21] of the multimode panel system where only the dynamic pressure increased, the parameter *X7-Y7* decreased gradually in a monotonic manner over the whole dynamic pressure range. In other words, this flutter boundary estimation would be more practical in a non-adaptive panel system. In such an adaptive system treated here, instability characteristics are considered certainly very complicated, because there seem to be multiple possibilities of flutter corresponding to different combinations of the modes coupled in the process of adaptation. As mentioned earlier, in the case of an ordinary aeroelastic system the dynamic pressure of the flow, i.e., the flow speed, is the unique variable, whereas, in this analysis, four spring constants, K_1 to K_4, are essentially variables, although all except for K_2 were kept at zero. In an actual system, these variables can not be treated separately.

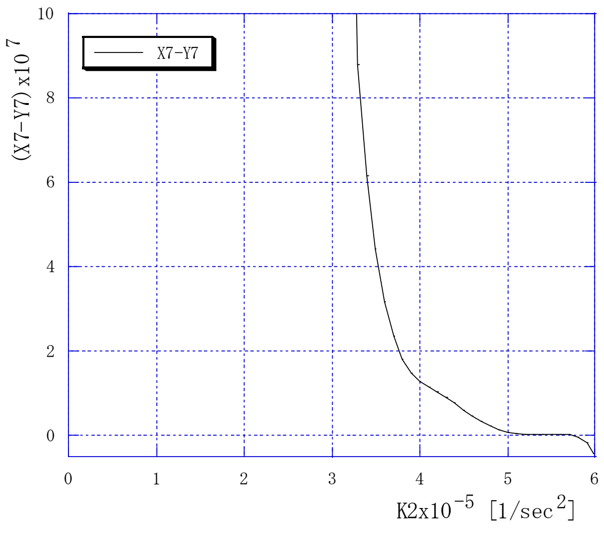


Figure 4. X7-Y7 versus K_2.

4. CONCLUSIONS

We have performed the preliminary study on flutter boundary estimation of the smart *adaptive* aeroelastic multimode system, using Jury's stability criterion. The instability was generated due to the coupling between the second and the third aeroelastic mode while the second-mode stiffness of the smart distributed spring increased as an adaptation. The numerical results showed that the prediction method founded on Jury's criterion was much more effective than the conventional damping approach. Further study is necessary to comprehend complicated behaviors of such adaptive aeroelastic systems in which multiple possibilities of aeroelastic coupling may exist in the process of structural adaptation.

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APPENDIX: Submatrixes $X_{i\text{-}1}$ and $Y_{i\text{-}1}$ used in Eqs. (15) are defined by

$$X_{n-1} = \begin{bmatrix} a(n+1) & a(n) & a(n\text{-}1) & .. & a(4) & a(3) \\ 0 & a(n+1) & a(n) & .. & .. & a(4) \\ 0 & 0 & a(n+1) & .. & .. & a(5) \\ .. & .. & .. & .. & .. & .. \\ .. & .. & .. & 0 & a(n+1) & a(n) \\ 0 & 0 & .. & 0 & 0 & a(n+1) \end{bmatrix} \qquad for \;\; n = 2,\ 4,\ 6,\ 8, \qquad (16.1)$$

$$Y_{n-1} = \begin{bmatrix} a(n-1) & a(n-2) & .. & .. & .. & a(2) & a(1) \\ a(n-2) & a(n-3) & .. & .. & a(2) & a(1) & 0 \\ a(n-3) & .. & .. & .. & a(1) & 0 & 0 \\ .. & .. & .. & .. & .. & .. & .. \\ a(2) & a(1) & 0 & .. & .. & 0 & 0 \\ a(1) & 0 & 0 & .. & .. & 0 & 0 \end{bmatrix} \qquad for \;\; n = 2,\ 4,\ 6,\ 8. \qquad (16.2)$$