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# Structural Design and Verification of a Vibrating Plate for a Piezoelectric Loudspeaker for Improved Performance

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## ABSTRACT

An effective structural design method to improve the frequency response of a piezoelectric loudspeaker is proposed in this paper. Typically, frequency responses of piezoelectric loudspeakers are characterized by sharp peaks near the resonance due to the high quality factor Q of the piezoelectric material used. However, the sharp resonant peaks limit the wider applicability of piezoelectric-based loudspeakers. One of the methods to get around this limitation is to excite multiple modes simultaneously of the vibrating plate. To explore this problem, we analyzed the relationship between the mode shapes and the sound pressure generated by a vibrating plate. As predicted by Raleigh's integral for sound radiation, it is confirmed that not all resonant modes of a vibrating plate leads to an effective sound pressure response in far field. We thus adopted a passive control to vary the vibrating behaviors of the plate. More specifically, we used structural design to remove and to restrain certain mode shapes so as to tailor the sound pressure. By choosing an effective structural design and selecting multiple modes by using Finite Element Method, we arrive at a much improved frequency response curve of the piezoelectric loudspeaker that is flatter to satisfy the requirements for a loudspeaker. It is believed that the applications of piezoelectric loudspeaker can be extended by using the method disclosed.

**Keywords:** piezoelectric loudspeaker, mode shape, frequency response, structural design.

## 1. INTRODUCTION

Frequency response of loudspeaker is usually the major the performance index and evaluation on fidelity of loudspeakers [1]. There are many kinds of loudspeakers available commercially nowadays, including coil driven loudspeakers, electrostatic loudspeakers, flat panel loudspeakers, and ribbon loudspeakers etc, each possesses distinguishing features and proper applications. Whereas the applications of loudspeakers in 3C industry demanding the convenience of portability, the image of high technology, compact and low-profile have becomeing ever more important terms of those features. Traditional coil driven loudspeakers is the most common loudspeaker nowadays, but it has the limitation on dimensions for its inherent driving structures. However, in addition to high potential and instant response, piezoelectric loudspeaker possesses the exact advantage of ease -miniaturization and low in profile . The only problem is the high quality factor of piezoelectric materials which cause steep frequency response piezoelectric loudspeakers on their resonant modes. Accordingly, piezoelectric loudspeakers are generally applied to buzzers which only blares a single frequency sound. Suppose the steep frequency response peaks of piezoelectric loudspeaker can be smoothen, the piezoelectric speakers can then be fit into the main stream of loudspeakers which is integrated in potable electronics.

The aim of this paper is to propose a structural design for a piezoelectric loudspeaker with improved frequency response. The vibrating behaviors are highly related to sound pressure level. However, most design and investigation of loudspeaker only considered the rigid body motion and eliminated the higher mode shapes which were considered as the disorder of vibration and could induce colorations of sound transmission [1, 2]. The relationships between the vibration behaviors and sound pressure level will be analyzed in this paper. The contributon of sound pressure level from every mode shapes will be invastigated.

The frequency response of the original circular plate are first determined by experiments and the mode shapes of the major peaks on the response curve were identifined. In the meantime, FEM analysis software COMSOL will be used to simulate the vibrations of the plates in order to verify the accuracy and dependability of FEM. Then, selection the mode shapes which are needed to be re-arranged or restricted by reinforcing the stiffness of particular region to alter the vibration behaviors which can improve the frequency response. Analyzing these design effects by using FEM to derive the optimum design, we then verify the results by experiments. The last part of the paper will discuss the design of array piezoelectric loudspeakers where we need to select the frequency by selecting the proper parameters by using FEM and then combine the above methods to analyze an array of loudspeakers to improve the ultimate performance of piezoelectric loudspeakers.

Furthermore, since the resonant frequency is related to dimensions and material characteristics, we choose the range from 700Hz to 6kH as the scope to improve for middle frequency range. The method proposed in this paper can certainly be applied to other frequency range and to select the main frequency range for improving the response.

## 2. THEORETICAL BACKGROUND

### 2-1. Plate Equations

In this paper, the vibrating plates are in circular shape. Here we take the polar coordinates r and θ for convenience, and consider a circular plate without in-plane forces and elastic foundations [3]. The differential equation of the plate is

$$\xi\nabla^2\nabla^2 w+\rho l\frac{\partial^2 w}{\partial t^2}=0 \tag{1}$$

Where

$$\begin{cases}\nabla^2=\dfrac{\partial^2}{\partial r^2}+\dfrac{1}{r}\dfrac{\partial}{\partial r}+\dfrac{1}{r^2}\dfrac{\partial^2}{\partial\theta^2}\\ \xi=\dfrac{\varsigma l^3}{12(1-\nu^2)}\end{cases} \tag{2}$$

, where $\rho$ is the density, $w$ is the deflection, $\varsigma$ is the bending or flexural rigidity, $l$ is the thickness of the plate, $\Upsilon$ is Young's modulus and $\nu$ is Poisson ratio.

From Eq. (1), we can derive the mode shapes of the circular plate [4] which is,

$$Z(r,\theta) = [C_1 J_n(\lambda r) + C_3 I_n(\lambda r)]\cos n(\theta+\phi) = C_1\left[J_n(\lambda r) - \frac{J_n(\lambda a)}{I_n(\lambda a)} I_n(\lambda r)\right]\cos n(\theta+\phi) \tag{3}$$

The node lines of the plate can be obtained by setting Eq. (3) to zero, which may be diametral lines or concentric circles. The values of *m* and *n* represent the number of diametral lines and concentric circles. A few mode shapes of circular plate are shown in Figure 1.

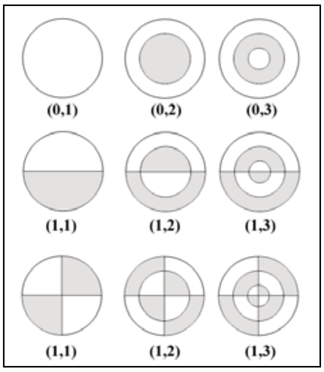



**Figure 1.** Mode shapes of circular plate.

### 2-2. Acoustics

As a simple baffled source, the far filed pressure [5, 6] can be written as

$$\mathbf{p}(r,t) = j\rho_0 \kappa (Q/\lambda r) e^{j(\omega t - kr)} \tag{4}$$

where $\rho_0$ is the density of medium, $\kappa$ is the speed of sound, r is the distance between source and pressure measured point, λ is wavelength, ω is angular frequency, k is wave number and Q is the real part of the complex source strength **Q**.

$$\mathbf{Q}e^{jwt} = \frac{\partial \mathbf{V}}{\partial t} = \int_A \vec{\boldsymbol{\eta}} \cdot \hat{n} dA \tag{5}$$

Where ***V*** is the complex volume displacement and $\vec{\boldsymbol{\eta}}$ is the complex velocity distribution of the source surface. For a pulsating sphere simple source, the complex source strength only has the real part.

For a very small vibrating plate δA, the pressure it contributes to the far field point will be,

$$\mathbf{p}(r,t) = j\rho_0 \kappa (|\eta_A| \delta A/\lambda r) e^{j(\omega t - kr)} \tag{6}$$

Where $\delta A$ is the area of this section and $\eta_A$ is surface velocity. By Rayliegh's first integral, the whole vibrating plate can produce far field pressure as,

$$\mathbf{p}(r,t) = \frac{j\rho_0 \kappa e^{j\omega t}}{\lambda} \int_A \frac{|\eta_A(r,t)| e^{-jkr}}{r} dA \qquad (7)$$

Let B be the amplitude of the displacement,

$$B_A(r,t) = Be^{j(\omega t - kr)} \qquad (8)$$

Then the velocity can be rewritten as,

$$\eta_A(r,t) = j\omega Be^{j(\omega t - kr)} \qquad (9)$$

So that Eq. (7) becomes

$$\mathbf{p}(r,t) = \frac{-\rho_0 \omega \kappa e^{j\omega t}}{\lambda} \int_A \frac{Be^{-jkr}}{r} dA \qquad (10)$$

In the discrete form

$$\mathbf{p}(r,t) = \frac{-\rho_0 \omega \kappa}{\lambda} \sum_{i=1}^{n} Be^{j(\omega t - kr_i)} \frac{\Delta A}{r_i} \quad \text{or} \quad \mathbf{p}(r,t) = \frac{-\rho_0 \omega \kappa}{\lambda} \sum_{i=1}^{n} B\cos(\omega t - kr_i) \frac{\Delta A}{r_i} \qquad (11)$$

When source is exciting by an harmonic force, the steady pressure has a distant of $r$ from the center of the vibrating plate would be,

$$\mathbf{p}(r,t) = \frac{-\rho_0 \omega \kappa}{\lambda} \sum_{i=1}^{n} B(x_i, y_j) \cdot \cos(\theta_{ij} - kr_{ij}) \frac{\Delta A}{r_i} \qquad (12)$$

Where $\theta_{ij}$ is the phase angle of the vibrating plate.

For describing sound pressures and intensities, it is very conventional to use logarithmic scales [7]. This is because humans feel the loudness between two sounds by their intensity ratio. Besides, logarithmic scale can compress the range of expressed numbers in view of the wide range of sound pressures and intensities.

Conventionally scale is sound pressure level (SPL). SPL depends on the measured pressure in the observing position.

$$SPL = 20\log(\mathrm{P}_{rms} / P_{ref}) \qquad (13)$$

Where $P_{rms}$ represents the root-mean-square sound pressure, $P_{ref}$ is a reference pressure and equal to $2*10^{-5}\ N/m^2$ in air. Here we can notice the coefficient of SPL is twice than $IL$ as a result of $I = \frac{P_{rms}^2}{\rho_0 c}$ in the progressions of plane and spherical waves. In the frequency response figure, Y-coordinate is for dB of sound pressure level and X-coordinate is for the frequency we excited.

## 3. EXPERIMENTAL SETUP

To observe mode shapes of the circular vibrating plate, we use carbon powders as the medium which will spread out when the plate vibrates and accumulating on nodel lines. We sprinkle carbon powders on the plate uniformly through a sieve and choose the frequency for the signals generated from the function generator and amplified by power amplifier. Finally, observe and photograph the plate to record mode shapes.

For analyzing the acoustic response of the plate, measuring the sound pressure level is needed. Standard microphone B&K 4190 is used to measure the sound pressure 1 meter away from the vibrating plate. The signals are generated from a NI-6250 DAQ (data acquisition) card which is controlled by a "LabVIEW" program. The pressure measured in an anechoic chamber by standard microphone will also be conveyed to DAQ and converted to decibel. The process of this measurement is illustrated in Figure 2.

From the above two measurements, we can observe the relationship between mode shapes and far filed sound pressure level.

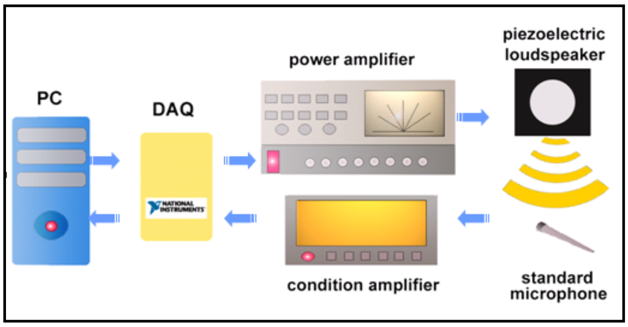



**Figure 2.** Experimental setup.

**Figure 3.** Measuring SPL in an anechoic chamber.

## 4. SIMULATION AND EXPERIMENTAL RESULTS

### 4-1. Frequency response and mode shapes of piezoelectric loudspeakers from experiment and FEM

In this paper we simulated the vibrating behavior of piezoelectric loudspeaker by FEM software COMSOL. The model of piezoelectric loudspeaker is shown in Figure 4 and the parameters of the simulation are showed in the APPENDIX. By choosing Eigen-frequency analysis, we can derive the resonant frequencies and their mode shapes, and then calculate the sound pressure level by integrating Eq. (12) and Eq. (13) where all the parameters can be derived from Harmonic Analysis.

The frequency response measurement of the piezoelectric loudspeaker is shown in Figure 5 and we can then observe mode shapes of those relatively strong and weak sound pressures to generalize the relationship between sound pressure levels and mode shapes. Furthermore, the simulated results are verified in Table 1.

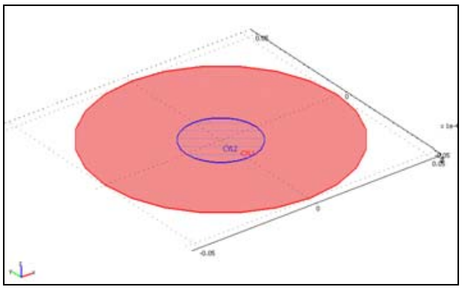


**Figure 4.** Model of piezoelectric loudspeaker.

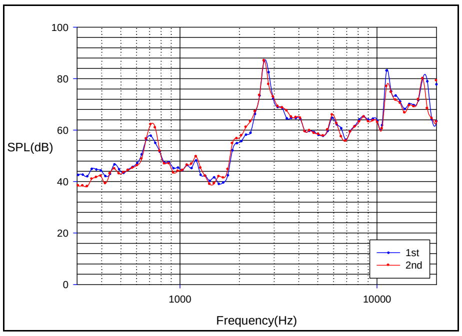
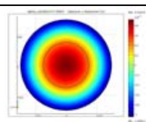
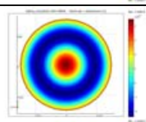
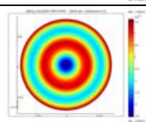
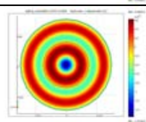
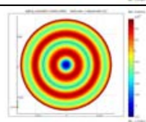



**Figure 5.** Frequency response of 60mm diameter brass plate.

**Table 1.** Mode shapes of the relative strong sound pressures of 60mm diameter brass plate.

| Brass plate, Diameter = 60mm | | | |
|---|---|---|---|
| Frequency | Experiment | Frequency (error) | FEM |
| 748Hz |  | 677Hz (-9.49%) |  |
| 2670Hz |  | 2564 Hz (-3.97%) |  |
| 5912Hz |  | 5983 Hz (1.2%) |  |
| 11166Hz |  | 10797 Hz (-3.3%) |  |
| 17060Hz |  | 16606 Hz (-2.66%) |  |

## 4-2. Restrained bars

From Eq.(12), we can know sounds would disappear for two situations, one is there is not enough vibrating amplitude which means the vibration frequency is off-resonant. The other case is that even though the plate is vibrating, it has symmetrical vibrating where the forward and backward pressures are exactly cancelled in far field. These two situations make the valleys in the acoustic frequency response curve that intensifies the unevenness of the curve.

On the other hand, sound would becomes strong when there is effective integration of far filed sound pressure. The mode shapes of vibrating plate which can produce effective sound pressures are found in the concentric circle form from both experiments and FEM analysis (Table 1). This property would appear in different materials or dimentions.

To smooth the acoustic frequency response of piezoelectric loudspeaker, we need to increase sound pressure of valleys and decrease peaks. Since we had confirmed that only the concentric circles mode shapes can produce the relative strong sound pressures. Reinforce the rigid factor can increase the difficulties of vibration where we can derive from Eq. (1). Adding bars in the radius direction on the plate would be an effective method to abate these concentric circle forms of vibration. Futhermore, this method can also destroy the symmetry mode shapes and increase the numbers of resonant frequency to increase the sound pressure of the valleys. The verification of mode shapes of the restrained bars added plate between experiment and simulation is showed in Figure 6. The improvement of piezoelectric loudspeaker with restrained bars is showed in Figure 7 where the acoustic frequency response curve becomes smoother.

Since this method alters the vibrating behavors by adding restrained bars, the rigidty ratio between vibrating plate and restrained bars or the arrangement and amount of restrained bars are very important. By comparing the average SPL and standard deviation, the former should have minimum loss and the later should have the maximum decrease, the optimum design can be derived in Table 2.

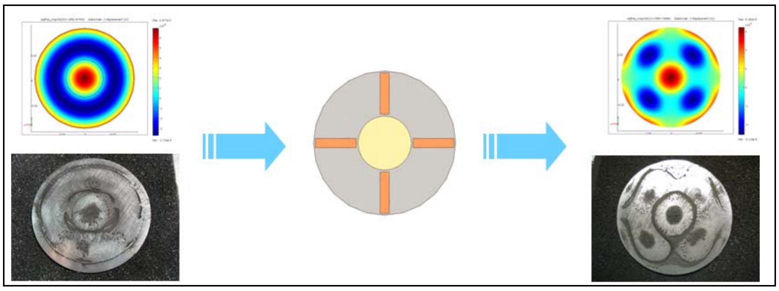


**Figure 6.** Effects of restrained bars.

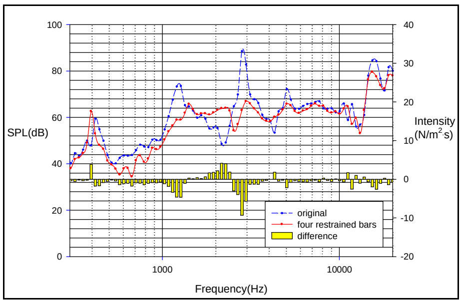



**Figure 7.** Frequency response of 100mm diameter steel plate with four restrained bars.

**Table 2** Effects of different type of restrained bars.

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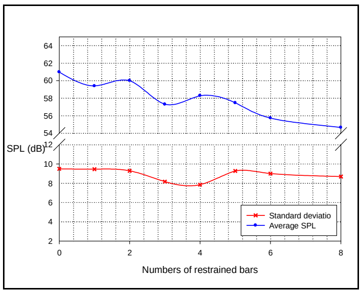
<td>
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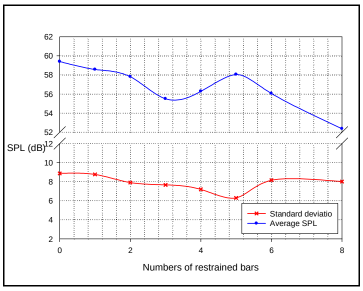
<td>
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<tr>
<td>Effects of different number of restrained bars on steel plate.</td>
<td>Effects of different number of restrained bars on brass plate.</td>
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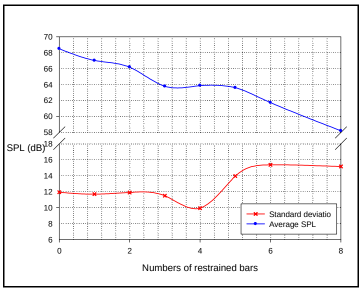
<td>
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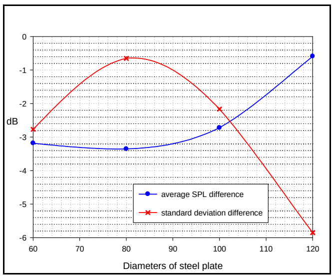
<td>
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<td>Effects of different numbers of restrained bars on aluminum plate.</td>
<td>Effects of four restrained bars on different dimensions steel plate.</td>
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## 4-3. Array loudspeakers

As adopting higher modes of vibration, there is one further method to improve piezoelectric loudspeaker. From Figure 5, we can notice the frequency responses of piezoelectric loudspeakers have many peaks and valleys. This phenomenon is considered defective in loudspeakers; but we can utilize the characteristics of piezoelectricity to achieve excellent performance by adopting array combination. Array loudspeakers will be a proper method since the peaks of each plates with different properties are staggered.

There are many properties would effect acoustic frequency response such as Young's modulus, Poisson ratio, thickness, and diameter. We choose diameter as the parameter in this paper. Firstly, we have to select the appropriate diameters whose frequency responses will be staggered. This part can be selected by simulating since FEM can effectively calculate the frequency response curve. Analyzing result is shown in Figure 8, from this figure we can find most of the 100 mm diameter steel plate's peaks are in the valleys of 120mm diameter steel plate, therefore 100mm and 120mm diameter steel plates are selected to be combined to form array loudspeakers.

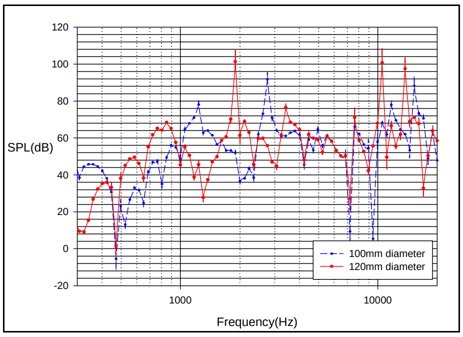

**Figure 8** Frequency response of combination of 100mm and 120mm diameter steel plates from FEM.

Then, **w**e integrate the restrained bars into array loudspeakers to observe the effect of improvement which is showed in Figure 10. The red curve illustrates that with the abatements of fluctuations and the complementary peaks of those two loudspeakers make the excellent improvement in piezoelectric loudspeakers. Finally we change the boundary conditions to foam tape for observing the feasibility in practical utilities, as in Figure 11 shows the performance had considerably improved.

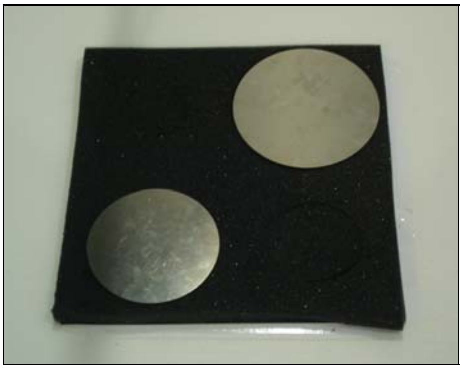
**Figure 9.** Array piezoelectric loudspeaker.

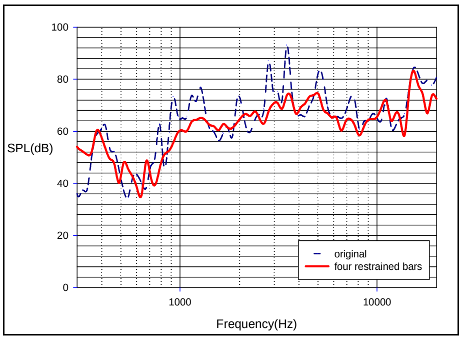

**Figure 10.** Frequency response of combinations of 120mm and 100mm diameter restrained steel plates.

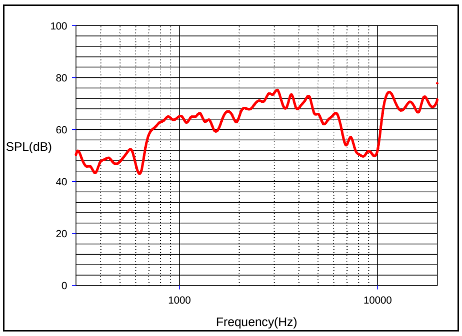


**Figure 11.** Frequency response of combinations of 120mm and 100mm diameter restrained steel plates at foam tape edges.

## 5. CONCLUSIONS

Through experiments and simulations, it can be concluded that passive structural design can effectively improve the acoustic frequency response of piezoelectric loudspeaker. The method proposed in this paper can efficiently decrease the standard deviation of a 120mm diameter steel plate from 12.28dB to 6.43dB within the frequency range 700Hz to 6 kHz while maintaining similar average sound pressure level. Furthermore, combine this method with array loudspeaker can further improve in piezoelectric loudspeaker which can make the standard deviation decrease to 5.52dB. This structural design method can not only decrease the peaks of the frequency response but also can increase the valleys. Since this method is applicable to all materials and dimension by selecting suitable restrained bars, the working frequency of this structural design piezoelectric loudspeaker can be varied by choosing different materials, thickness, or diameters etc. These properties can be selected by calculating frequency response by FEM to estimate the effect of structural design.

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## APPENDIX

| Property / Material | Young's modulus [GPa] | Poisson ratio [1] | Density [kg/m$^3$] | Thickness [mm$^2$] | Diameter [mm$^2$] |
|---|---|---|---|---|---|
| Stainless steel | 210 | 0.28 | 7800 | 0.3 | 60~120 |

| | | | | | |
|---|---|---|---|---|---|
| Brass | 102 | 0.33 | 8300 | 0.3 | 60~120 |
| Aluminum | 72 | 0.34 | 2700 | 0.3 | 60~120 |
| PZT | Density: 7400 kg/m$^3$<br>Relative permittivity: $\begin{bmatrix} 3130 & 0 & 0 \\ 0 & 3130 & 0 \\ 0 & 0 & 3400 \end{bmatrix}$<br>Coupling matrix: $\begin{bmatrix} 0 & 0 & 0 & 0 & 7.41 & 0 \\ 0 & 0 & 0 & 7.41 & 0 & 0 \\ -2.74 & -2.74 & 5.93 & 0 & 0 & 0 \end{bmatrix}$ e-10[C/N]<br>Compliance matrix: $\begin{bmatrix} 1.65 & -0.478 & -0.845 & 0 & 0 & 0 \\ -0.478 & 1.65 & -0.845 & 0 & 0 & 0 \\ -0.845 & -0.845 & 2.07 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4.35 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4.35 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4.26 \end{bmatrix}$ e−11[1/Pa] | | | | |