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Design of smart lightweight structures: Simultaneous optimization of mechanical and shunt circuit parameters

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Abstract

Modern passenger cars are increasingly restrained to lower CO_2 emissions and therefore to reduce weight. In order to accomplish that, the replacement of metals by lightweight composite materials is a viable solution. However, this can significantly change the NVH behavior of the vehicle, thus requiring new design techniques. One well-known technique uses shunted piezoelectric transducers, which are applied to a mechanical structure and connected to an electronic circuit. When correctly designed, the shunt can significantly attenuate the vibration of the system without excessive mass addition. Much research has been done in the last decades to analyze various types of shunts in terms of their potentials and characteristics. Nevertheless, they are not yet integrated in many technical structures, not only because of high costs, but also due to the high effort to design the electronics effectively together with the mechanical structure and transducer. This paper presents a methodology for a global design of a smart structure, instead of isolated sub-systems, where different functions of the electromechanical system (host structure, actuators and electronics) are simultaneously optimized. The design optimization focuses on numerical and experimental analyses of composite materials, with bonded piezoelectric ceramic transducers and connected to a semi-active shunt network, more precisely a resistor-inductor circuit with a negative capacitance. Here in particular, the negative capacitance has to be built through a synthetic circuit (negative impedance converter). It is based on an operational amplifier, a fact that increases the number of variables in the optimization process and makes the real values diverge from the theoretical ones. From the mechanical side, the material layup and the part geometry are the main design variables, whereas for the actuator, its position and dimensions are crucial. As an application example, a composite part together with a piezoelectric actuator is improved by parametric optimization, regarding its mass, its static and dynamic behavior. At this phase, the generalized electromechanical coupling coefficient (GEMCC), which describes the modal energy transfer between the mechanical and the electrical systems, is one of the main design variables, since it dictates the performance of the semi-active vibration control. Finally, the parameters of the electronic shunt network are optimized to minimize the mechanical response of the structure.

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1. INTRODUCTION

Since the 1990s, there has been an enormous growth in smart structures technology. Several fields of study benefit from this development, such as space vehicles, aircrafts, railway and automotive systems, robots, heavy machinery, medical equipment, etc. A wide range of applications include noise and vibration suppression, damping increase, structural health monitoring, energy harvesting, etc. A smart structure typically consists of a host structure, actuators and sensors, a microprocessor that analyzes the signals, a control law to change the characteristics of the structure and integrated power electronics. More importantly, a smart structure has the ability to adapt its properties according to external stimuli in a controlled manner. Several types of structures, actuators, sensors and control laws have been studied and published. Potential applications for smart structures are presented in [1].

In this paper, investigations are targeted at composite materials. In the context of smart structures design, they are often a reasonable choice, since they present a good stiffness to weight ratio. Fiber reinforced composites allow more possibilities than commonly used materials and their custom designed nature offers a unique opportunity to minimize weight, embed actuators and improve dynamic behavior.

The main problems that appear when designing a lightweight structure, however, can be related to high vibration levels, due to low mass and damping inherent to composite materials. In order to attenuate this undesirable effect, passive methods can be used, e.g. mechanical tuned mass absorbers, but they are often associated with a high mass added onto the structure. Active vibration control, on the other side, can effectively reduce vibrations, but there is always the need for actuation power and relatively high demand for computational controller performance.

Between these two techniques are situated the semi-passive and the semi-active methods, which benefit from low cost components and a simple controller design. These techniques consist of applying a piezoceramic transducer to a mechanical structure and then connecting it to a shunt circuit. In that way, it is possible to change the mechanical properties of the complete system without the need of actively driving the actuator. Various types of shunts have already been developed and tested regarding their potentials and characteristics in vibration control. However, they are not yet integrated in many technical structures, due to the high effort to design a semi-active device, since there are many correlated parameters that have to be optimized in order to get an applicable solution.

In this context, it has already been evidenced in the literature [2] that optimizing an actuator on pre-determined host structures, and vice versa, is little effective. More importantly, when considering shunting techniques, the GEMCC is the driving parameter, so its optimization is necessary. A number of analytical or numerical methods for optimizing smart structures have been published [3] [4] [5], but metals and academic geometries are usually used, only active control is considered and no experimental data is used in the design loop. The work presented in this paper is mainly interested in numerical results as design guidelines, but the idea of taking into account measured data throughout the optimization process is already introduced.

2. SMART LIGHTWEIGHT DESIGN

In order to show that shunting techniques can be applied to technical structures, this study will focus on a preliminary composite part, which is depicted in Figure 1. It will be used as a starting point for the simultaneous optimization carried out later. It consists of a cantilever CFRP beam with I-shaped cross-section, thin wall and hollow interior. Its geometry and dynamic characteristics represent a vehicle front suspension component, the control arm. The tip mass vibrates in the vertical direction and represents the wheel connected to the front suspension. Additionally, piezoceramic transducers are bonded to the beam's external surface so as to convert mechanical into electrical energy and enable semi-active vibration control.

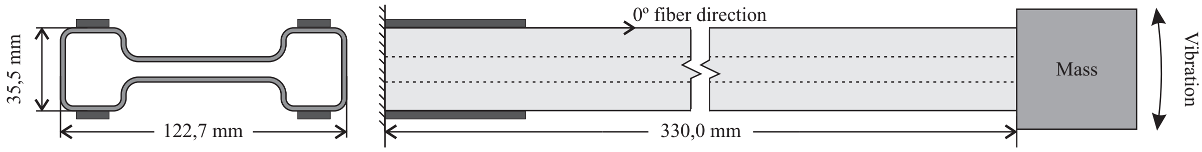

Figure 1. Preliminary cantilever CFRP I-beam with bonded piezoceramic transducers

2.1 Composite structure modeling

The composite structure was modeled as shell finite element (FE) using ANSYS Composite PrepPost software. It is based on the classical laminate theory and facilitates the analysis of layered composite structures using numerous variables. Composites provide stiff and lightweight design, with enough flexibility for complex shapes, but it often requires high efforts to define input variables in the optimization. In this sense, the engineering of a composite product is an iterative process. This involves stress and deformation evaluation and, in the case of an insufficient design or material failure, the geometry or laminate has to be modified and the evaluation is repeated.

For this reason, when optimizing a composite part, a compromise must be found between several input and output variables, since a mathematical global optimum might be difficult to attain. Among the input variables there is the part geometry, the fiber material (carbon or glass), the type of layer (unidirectional or woven), the thickness and number of layers, layup sequence, ply orientation, etc. Output variables include mass, resultant static stiffness, stress and strain levels, failure criteria, etc. Moreover, the manufacturing process should be kept in mind during the design process, in order to keep production complexity and costs low. In the case of a smart structure, where an actuator will be included afterwards, it is also important to analyze the position of the piezoceramics in advance in order to improve strain levels and to optimize the GEMCC.

2.2 Piezoceramic transducers

The term piezoelectricity refers to the effect present in many natural crystals that is the generation of electricity under mechanical pressure. It was first observed by the Curie brothers in 1880 but had no practical application until the First World War, when it was used in ultrasonic emitters. With the discovery of piezoceramics exhibiting better piezo effect than natural materials, like the lead zirconate titanates (PZT), a large scale manufacturing became possible and the application in adaptive structures grew.

In the last decades, piezoceramics became the major type of actuator being investigated for smart structures. Much research has been done trying to efficiently put together metal structures and surface bonded actuators, but few technical applications can be found where piezoceramic materials and composite structures are designed together. Still several considerations have to be taken into account when designing such systems, for example the GEMCC, the load carrying capability, durability, manufacturing techniques, etc.

In the FE model, the linear constitutive equation is embedded in a piezoelectric 20-node solid element. The material characteristics for the piezoceramic PIC151 (compliance, piezoelectric strain and permittivity matrices) are provided by the manufacturer. This allows the analysis of mechanical and electrical DOF in order to define input and output variables during the optimization process.

In a first approach, using the preliminary composite structure, a study regarding the piezoceramic size, position and boundary conditions has been carried out. First, the location of the maximum strain level in the structure for the eigenfrequency of interest has been identified. Four rectangular piezoceramics have been hence placed on the outer surface and near the clamp. They are electrically connected in parallel so that only one shunt circuit will be used.

As already mentioned, an important parameter in designing smart structures with piezoceramic transducers and semi-active vibration control is the GEMCC. It describes the energy transfer between the mechanical and the electrical systems at a given eigenfrequency. It can be calculated by Eq. 1 once the eigenfrequencies of the structure with open and short circuited electrodes are known.

$$K_{31}^2 = \frac{\omega_{\text{open}}^2 - \omega_{\text{short}}^2}{\omega_{\text{short}}^2} \qquad (1)$$

A sensitivity analysis showed that the thickness of the piezoceramic influences the GEMCC more than its width and length, so it has been defined as the main input variable. It has also been shown that there is one optimal value for the thickness that maximizes the GEMCC.

3. SEMI-ACTIVE VIBRATION CONTROL

Semi-active vibration control techniques take advantage of electromechanical transducers, like piezoceramics, which are capable of converting mechanical energy into electrical energy and vice versa. In a structure incorporating a piezoceramic, its stiffness acts in parallel with the stiffness of the host structure. Hence, by connecting it to a shunt circuit, its mechanical impedance can be controlled and the dynamic response can be improved. In the context of this paper, the shunting technique will be used to increase damping in systems with low inherent structural damping, hence the shunt damping term.

The most basic electric circuit to reduce vibrations with a shunted system is the purely passive resistive shunt (R-shunt). Additionally, [6] introduced the resistive resonant shunt (RL-shunt), in which an inductor is connected to the piezoceramic and the resistor. Since the piezoceramic electrically behaves as a capacitor, the resultant RLC circuit is a damped resonant system, which can be tuned to a certain frequency (e.g. the eigenfrequency of the mechanical system) and can therefore perform similarly to a mechanical tuned mass damper. The optimal resistance and inductance values, for which the electrical energy dissipation is maximized, and therefore the mechanical displacement is minimized, have been derived. It is also stated by [6] that the optimal values highly depend on the GEMCC.

3.1 RLC-shunt circuit

A method of artificially increasing the GEMCC and partly getting rid of its dependency has been investigated in [7]. It is suggested that the use of a negative capacitance, together with an RL-shunt, highly improves vibration attenuation, a fact that has been shown both analytically and experimentally. When a series negative capacitance is connected to a mechanical system, it behaves as a spring element with a negative stiffness, thus reducing its eigenfrequency. Moreover, if a negative capacitance is inserted into an RL-shunt, the two arising poles, characteristic for the absorption effect of the RL-shunt, spread away from each other, making this application efficient in a broader frequency range. Another advantage is that the needed optimal inductance value is much smaller compared to the value of a pure RL-shunt. It can be therefore enough to use a physical coil instead of a synthetic inductor, which can simplify circuit design.

In this study, focus will be given to the RLC-shunt, in which a resistor, an inductor and an additional capacitor are connected to the piezoceramic. At last, the shunt using a negative capacitance is defined as semi-active, since it transmits external energy in form of actuation forces to the host structure, still without the need of sensor information.

3.2 Negative impedance converter (NIC)

Since there is no passive element with a negative capacitance value, one possible technique for obtaining such an element is to use a synthetic impedance. The circuit shown in Figure 2 belongs to a general class of circuits known as negative impedance converter (NIC) and has been first applied by [8] in the context of shunt damping. This relatively simple circuit is based on an operational amplifier and performs a signal inversion of a passive element, in this case, a capacitor.

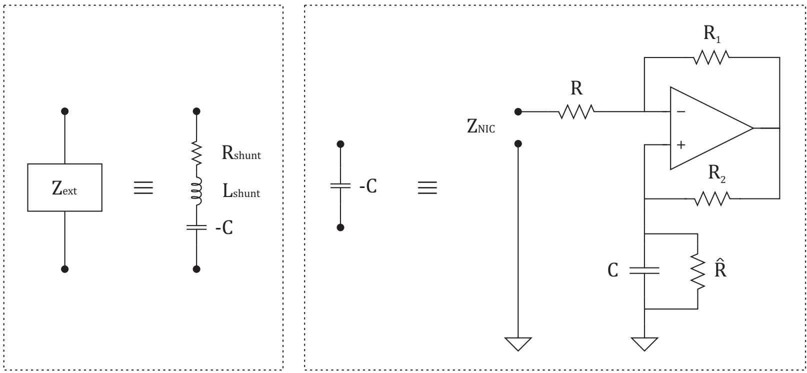

Figure 2. Series RLC circuit with a negative capacitance and its circuit implementation

Considering an ideal op-amp, it can be assumed that the impedance at the terminals is described by Eq. 2, as suggested by [9]. When a limited frequency band and a limited input voltage are considered, it can also be assumed that this impedance approximately represents an ideal negative capacitance.

$$Z_{NIC}(s) = \frac{Rs + (1/\hat{R}C)(R - \hat{R}R_1/R_2)}{s + (1/\hat{R}C)} \approx -\frac{1}{sC} \tag{2}$$

A negative capacitance can significantly improve the performance of an RL-shunt. However, if not tuned correctly, it can destabilize the system. The main advantage of the RLC-shunt compared to the RL-shunt is that the maximum attenuation depends very little on the GEMCC. Nevertheless, the disadvantage of a small GEMCC is that the optimum negative capacitance value will be very close to the stability boundary.

Several studies have been carried out using the NIC with shunt damping, but robust circuit design guidelines are yet to be explored. The work developed in [9] uses a closed-loop transfer function analysis to show that the circuit parameters must be chosen in a certain way to obtain stability for the complete electromechanical system. However, precise conditions for all parameters are not derived and the final tuning is still done empirically.

It is important to keep in mind that the mechanical structure, together with the piezoceramic and the shunt circuit, contribute with poles and zeros to form a single control loop. The overall performance and stability depend on each of them, so an understanding of the global system is crucial when designing a smart structure with shunt damping.

4. EQUATION OF MOTION

Based on the single degree of freedom equation of motion that has been analytically derived in [10], it is possible to generalize the circuit that is shunted to the transducer through an electrical impedance, as can be seen in Figure 3.

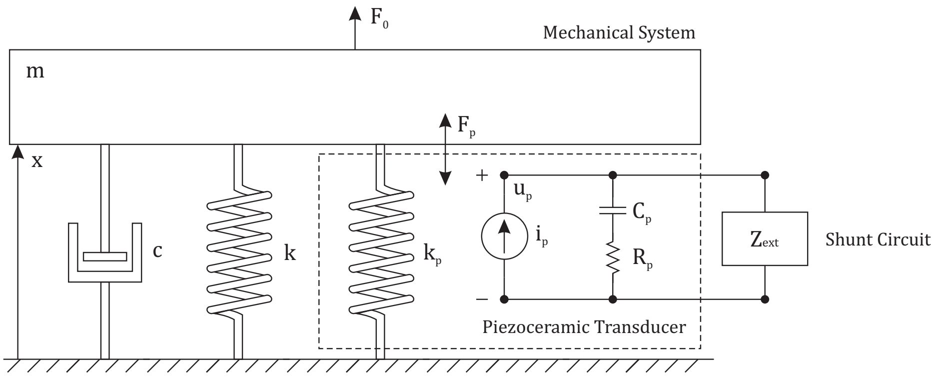

Figure 3. 1-DOF shunted electromechanical system

Considering a forced vibration of a mass *m*, which represents the equivalent vibrating mass of the composite beam, piezoceramic transducers and tip mass combined, connected to a spring *k*, a damper *c* and a piezoelectric element, which also has a mechanical stiffness k_p, the motion of the mass reads:

$$m\ddot{x} + c\dot{x} + kx + F_p = F_0 \tag{3}$$

Furthermore, F_p is the force generated by the piezoelectric element, which can be derived from the constitutive electromechanical equations of a linear piezoelectric material:

$$F_p = k_p x - k_p d_p u_p \tag{4}$$

where u_p is the voltage generated by the piezoelectric element and d_p is the charge density per unit stress of the electromechanical system, as defined by [10], which can be obtained using:

$$d_p = \frac{1}{k_p}\sqrt{K_{31}^2 C_p (k + k_p)} \tag{5}$$

The piezoceramic transducer electrically behaves as a current source i_p connected in parallel to a series RC-circuit of values R_p and C_p. The electric current generated by the transducer can be calculated by:

$$i_p = -k_p d_p \dot{x} \tag{6}$$

Considering the external shunt circuit, here represented by the impedance $Z_{ext}(s)$ in the Laplace domain, as a series RLC-circuit, where C is substituted by a negative impedance converter, $Z_{ext}(s)$ reads:

$$Z_{ext}(s) = R_{shunt} + sL_{shunt} + Z_{NIC}(s) \tag{7}$$

Since the external impedance is connected in parallel to the internal RC-circuit, the equivalent impedance $Z_{eq}(s)$ seen by the current source is given by:

$$\frac{1}{Z_{eq}(s)} = \frac{1}{Z_{ext}(s)} + \frac{1}{R_p + 1/sC_p} \tag{8}$$

When combining Eq. (6) and Eq. (8), it is possible to obtain the final voltage generated by the transducer:

$$U_p(s) = Z_{eq}(s) I_p(s) = -Z_{eq}(s) k_p d_p X s \tag{9}$$

Considering the displacement *X* of the mass *m* and the voltage U_p as variables in the Laplace domain, substituting Eq. (4) into Eq. (3) and using Eq. (9), the dynamic behavior of the system is finally given by the following electromechanical equation of motion:

$$\begin{bmatrix} m & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} X \\ U_p \end{bmatrix} s^2 + \begin{bmatrix} c & 0 \\ Z_{eq}k_p d_p & 0 \end{bmatrix}\begin{bmatrix} X \\ U_p \end{bmatrix} s + \begin{bmatrix} k + k_p & -k_p d_p \\ 0 & 1 \end{bmatrix}\begin{bmatrix} X \\ U_p \end{bmatrix} = \begin{bmatrix} F_0 \\ 0 \end{bmatrix} \quad (10)$$

5. EXPERIMENTAL ANALYSES

Before a simultaneous optimization takes place, it is very important to carry out some experimental analyses to fit the numerical model in order to correctly assess all mechanical and electrical parameters of the preliminary structure.

Initially, an experimental dynamic analysis of the sole composite part was performed. This study is mainly interested in investigating the first eigenfrequency, which represents a bending mode. The acceleration per unit force (inertance) was measured at the tip mass. The strain in the longitudinal direction was measured using a resistive strain gauge at the midpoint of the location where the piezoceramic transducers would be bonded.

Once the transducers were applied to the structure, a second experimental analysis was made. Again, the inertance was measured at the tip mass, when the electrodes of the piezoceramics were either open or short circuited, so as to allow the calculation of the GEMCC. The dynamic voltage generated by the transducers per unit force was also measured. The dynamic measurements were made using an FFT analyzer, an impedance head and an electromagnetic shaker.

Additionally, the piezoceramics internal capacitance and series resistance values were measured using an impedance analyzer. The values were obtained by fitting the impedance curve of an equivalent circuit model of the electromechanical structure. These values can also be approximately obtained by a single measurement made at a higher frequency than the first resonance, for example at 120 Hz. The main experimental results can be found in Table 1.

Table 1 Experimental results of the cantilever CFRP I-beam with bonded piezoceramic transducers

Description	Symbol	Value	Unit
Open circuit mode	f_{open}	62,52	Hz
Short circuit mode	f_{short}	62,08	Hz
GEMCC	K_{31}	0,12	—
Piezoceramic capacitance	C_p	36,4	nF
Piezoceramic resistance	R_p	17	Ω
Damping Ratio	ζ	0,93	%

5.1 Model update

After analyzing the preliminary composite structure without and with the piezoceramic transducers, the experimental results were used to update the FE model. This is done in order to have a precise representation of the real smart structure and to achieve more robust results during the further numerical optimization. The experimental setup and the FE model of the structure can be seen in Figure 4.

First, since the support of the real composite cantilever beam does not represent an ideal clamp, an elastic coefficient was included in the FE model between the composite material and the steel support. This coefficient has been chosen so as to fit the calculated and measured inertance curves around the first eigenfrequency. The experimental results and the fitted analytical results using the electromechanical equation of motion can be seen in Figure 5.

Additionally, it can be noticed that the calculated and measured strain are in relatively good agreement. The small error between the curves might come from the fact that a shell model was used to represent the beam, instead of a solid model, which would increase the number of nodes and therefore the calculation time, which is not desirable when performing an optimization.

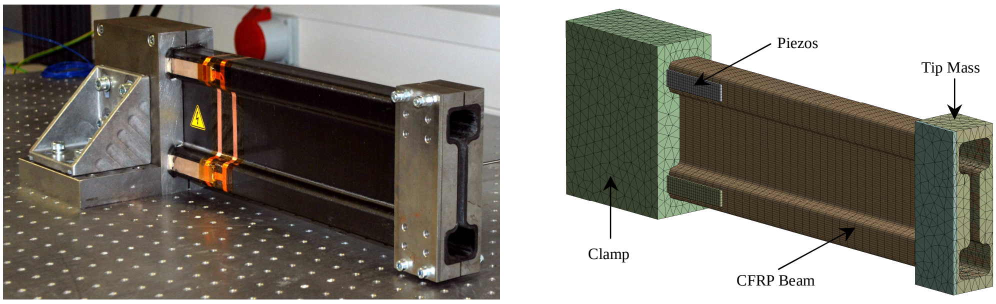

Figure 4. Experimental setup and FE model of the smart structure

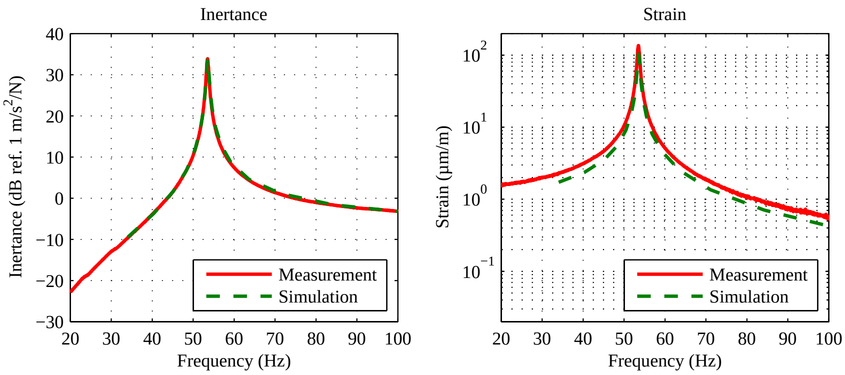

Figure 5. Analyses results for the sole cantilever CFRP I-beam

Secondly, the composite part with the bonded piezoceramic transducers was analyzed.

In order to better represent the bonding of the transducers to the composite structure, an elastic bonding coefficient was introduced in the contact regions. The coefficient was found by fitting the calculated GEMCC to the value found in the experimental analysis.

Another very important parameter in the design process of a semi-active system is the capacitance value of the transducers. When not correctly calculated, the shunt circuit will not be tuned and its optimization will lead to a wrong mechanical displacement of the structure. In the FE model, the driving parameters for the capacitance are the dimensions of the transducers and the permittivity of the piezoceramic material, which depends on the boundary conditions and is only known at two conditions given by the manufacturer (free and clamped). Since in a real structure the transducers are neither clamped nor free, the permittivity has been defined as a design parameter allowed to vary. Its value has been adjusted to fit the measured capacitance of the piezoceramics at a given frequency and the dynamic voltage response.

The measured and simulated inertance for both open and short circuit configuration can be seen in Figure 6. The dynamic voltage per unit force is depicted in Figure 7. It is noticeable that the model description depicted in Figure 3 and the electromechanical equation of motion describe with a very good precision the dynamic behavior of the first eigenfrequency of this smart structure.

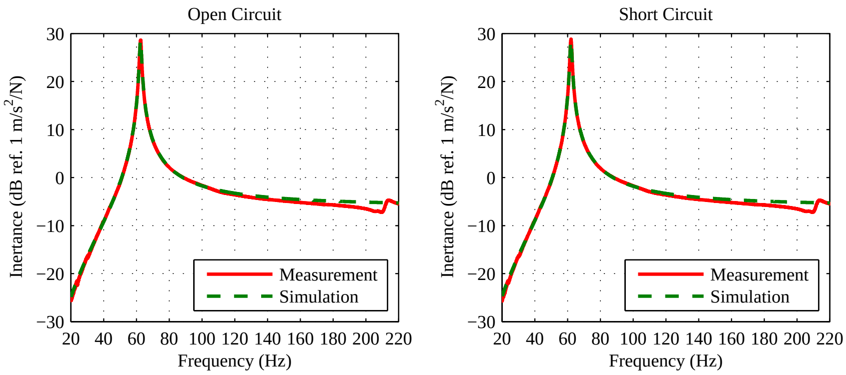

Figure 6. Inertance of the cantilever CFRP I-beam with bonded piezoceramic transducers

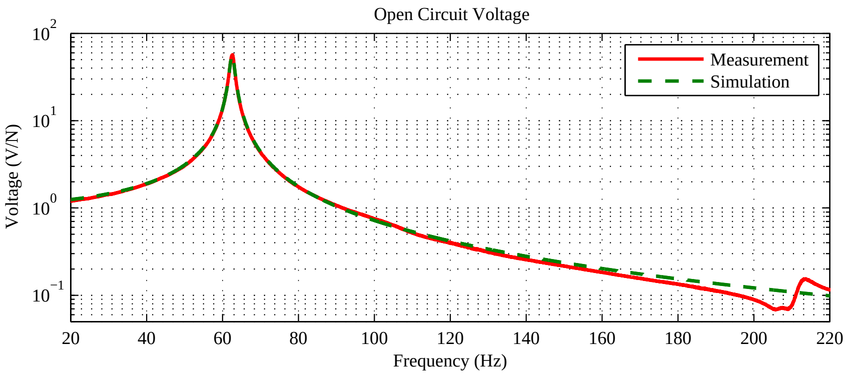

Figure 7. Dynamic voltage response per unit force

6. SIMULTANEOUS OPTIMIZATION

Once a representative and trustworthy model of the preliminary composite structure with bonded piezoceramics is obtained, it can be further refined together with the electronics in order to achieve a given set of specifications. In one iteration loop only, it is possible to define input parameters for the composite structure, the transducers and the shunt circuit, so that they are simultaneously optimized for common objectives.

For the composite structure, some fixed parameters were defined previously to the optimization itself, after a sensitivity analysis. The fiber material was chosen to be carbon, due to a high stiffness to weight ratio. The type of layer was chosen to be unidirectional, since it allows a better distribution of strain among the fibers, which is desirable when placing the piezoceramic transducers. The layup sequence was chosen to be 0° / ±45° / 90° with a commercially available ply thickness (0,23 mm). It is assumed that a 90° layer on the external surface can increase the strain under the piezoceramic and eventually enhance the GEMCC. A 0° layer on the internal surface guarantees the stiffness of the part. A ±45° layer in between makes a smooth transition and prevents delamination. Among the input variables for the composite structure, there are the cross-section dimensions and the number of layer for each orientation.

For the piezoceramic transducers, the thickness has been shown to be the most influential parameter, so it has been defined as a variable in the optimization process. The width was limited by the geometry of the beam's cross-section, so it has been fixed at 15mm. The length was fixed at 70 mm, the maximum value the manufacturer is able to reach.

For the shunt circuit, the component values in the oscillating circuit (R_{shunt}, L_{shunt} and –C) were defined as input variables. For a give mechanical structure, i.e., *m*, *c*, *k* and k_p fixed, it has already been shown by [10] that there is an optimal set for these values that minimize the mechanical displacement X of the structure. Furthermore, the component values in the NIC circuit (R, R1, R2 and $\hat{R}$) were also defined as input variables. The capacitance in the circuit was fixed at 68 nF. Regarding the NIC circuit, it has been shown by [9] that there are a certain number of rules to be respected to guarantee stability for the complete smart structure. These rules are respected and the stability of the global system is verified at each iteration step.

After defining the input variables, a certain number of design points are calculated using the FE model in order to create the design space. Using the previously derived equation of motion, it is therefore possible to find a set of parameters for the mechanical structure, the piezoceramic transducer and the shunt circuit that simultaneously minimize the mechanical displacement of the structure.

The optimal space-filling design was used to obtain the design points in order to achieve a more uniform space distribution inside the boundary values. Afterwards, the response surface was created by interpolating the results using a neural network algorithm.

The final objectives of the optimization are obtained using the multi-objective genetic algorithm and were such as 1) to fit the static stiffness of the complete smart structure to a certain pre-defined value, 2) to maximize the GEMCC and 3) to minimize the inertance around the first resonance peak.

6.1 Numerical results

In order to evaluate the potential of the preliminary structure with shunting techniques, an RLC-shunt circuit has been numerically optimized and introduced in the equation of motion. The result is shown in Figure 8. It can be seen that, when correctly designed, the shunt circuit with a negative capacitance has the potential to reduce 30 dB of the vibration level around resonance.

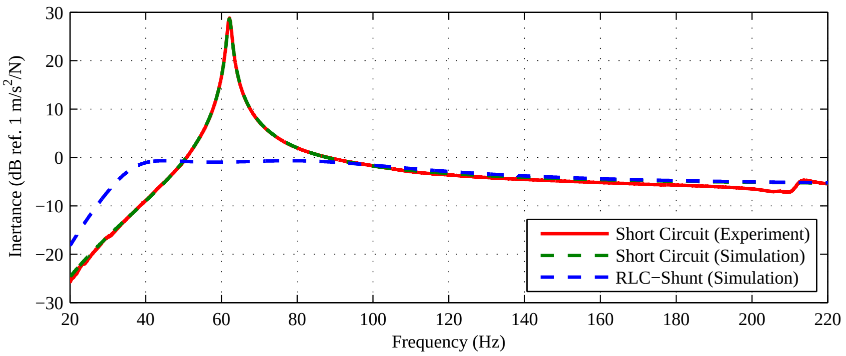

Figure 8. Experimental and simulation results of the smart structure without and with shunt

Next, when the composite structure is optimized together with the piezoceramic transducers, a slightly different geometry is obtained in comparison to the preliminary structure. It can be seen in Figure 9 that the composite structure has now a thinner wall than the preliminary studied case, since part of its stiffness has been replaced by the piezoceramic transducers.

A mass reduction can also be potentially achieved when optimizing both systems together, with a further advantage that the structure now has an integrated function.

The GEMCC, on the other hand, has remained the same as before, even though it was set to be maximized in the optimization. This does not constitute a problem for the semi-active vibration control, since the same level of vibration reduction can be achieved.

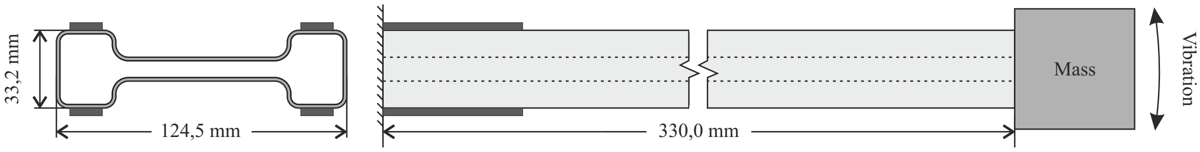

Figure 9. Simultaneously optimized CFRP beam with transducers

7. CONCLUSIONS

To sum up, the simultaneous optimization process of a smart structure can be represented in a flowchart diagram, as depicted in Figure 10. Initially, a preliminary composite beam has been chosen as a case study that stands for a real structure. Then, its dynamic characteristics have been analyzed through experimental tests and further used to fit a numerical model. Afterwards, piezoceramic transducers have been bonded to the structure and the measured electromechanical properties have again been used to adjust a numerical model, which at this stage is reliable and can be used for further optimizations. After that, an RLC-shunt circuit was integrated in the design loop through an electromechanical equation of motion. Finally, a simultaneous optimization of structural parameters, transducer dimensions and circuit component values have been shown to be efficient in minimizing the mechanical response of the structure.

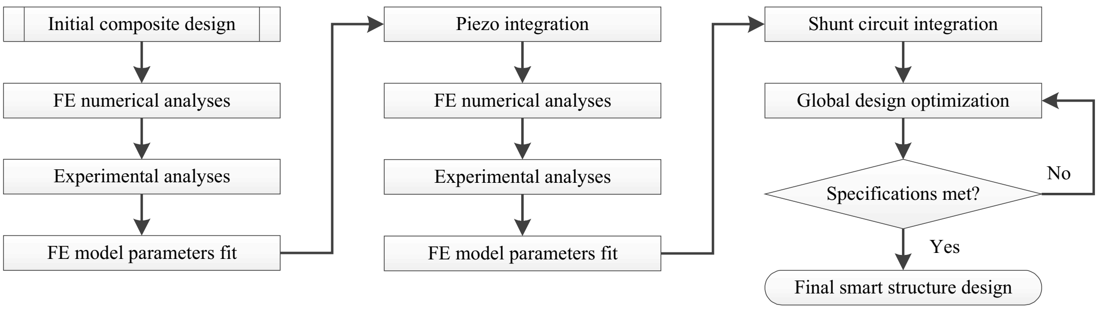

Figure 10. Smart structure optimization flowchart

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