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Robustness of Multiple Piezoelectric Vibration Absorbers

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Abstract

The paper aims to reveal an effect of multiplexing shunt circuits composed of inductors and resistors connected to piezoelectric transducers in order to improve a robustness of a PVA (Piezoelectric Vibration Absorber). The PVA is well known effective method to suppress a vibration of an adaptive structure, and its weakness is low robustness against to the change of dynamic parameters of the system including the main structure and the absorber. To improve the robustness, this paper proposes multiple PVAs that compose of partialized piezoelectric transducers and several shunt circuits. The optimization problems to determine the frequencies and the damping ratios of the multiple PVAs are multi-objective problems, and are solved by the real-coded genetic algorithm in the paper. A cramped beam of Aluminum patched with four groups of piezoelectric ceramics was treated in simulations and experiments. Numerical simulations revealed that the multiple PVAs designed by the proposed method had twice wider tolerance of the change of the natural frequency of the main structure. Furthermore, experiments using a thermostatic bath were conducted to reveal the effectiveness and the robustness of the multiple PVAs. The experimental results showed that the multiple PVAs are more robust than the single PVA under the variable temperature environment. In conclusion, multiple PVAs are effective and robust vibration control method for adaptive structures.

1. INTRODUCTION

The paper aims to reveal an effect of multiplexing shunt circuits in order to improve a robustness of a PVA (Piezoelectric Vibration Absorber) that is a damping device composed of a piezoelectric transducer connected with an inductor (L) and a resistor (R). PVA is a analogy of a dynamic vibration absorber (DVA).

DVA[1] was invented by Hermann Frahm in 1909. Frahm's DVA is usually illustrated as an additional system of a single mass and a single spring. Its natural frequency is set to be closed to that of the main system. This sort of DVA works well when the main system is excited at its natural frequency, however it does not work well in other cases. In 1928, Ormondroy and Den Hartog showed that a new DVA with a damper is more efficient than Frahm's DVA when the frequency of excitation is on broadband. They also proposed an optimal tuning method that is known as the "Fixed Point Theorem."

Since the advent of this DVA, many researchers and engineers have studied various passive, active, and semi-active DVAs. Today, the DVA is the most important vibration control device. Finding an optimal tuning point is a fundamental issue in DVA research. The many ways that have been proposed fall into three categories depending on estimated indicators. 1) Minimization of the H_∞ norm of the transfer function. 2) Minimization of the H_2 norm of the transfer function. 3) Maximization of the minimum modal damping ratio. The first indicator is used for damping periodical responses of the main system. The second one is used for damping random responses. The third one is used for damping impulse responses. Recently, analytical solutions have been obtained for these optimization problems in regard to a single DVA in a single degree of freedom (SDOF) system[2].

Multiple DVAs that contain several DVAs of which natural frequencies are set on around the natural frequency of main structure. This method improve the effectiveness of DVA without mass increasing. Zuo et al.[3,4] proposed an optimal tuning method for multiple DVAs in a multiple degree of freedom (MDOF) system by a numerical calculation. Their methods are based on the calculation techniques used for optimal feedback control theorems such as the H_2 optimal control theorem and the H_∞ optimal control theorem. The method calculates the stiffnesses and damping coefficients of several or MDOF DVAs attached to the MDOF system.

Piezoelectric materials are often used for adaptive structures to suppress vibration. It is well known that piezoelectric material shunted to a LR circuit behaves like the single DVA[5]. Various researches have dealt with the modified PVAs[6,7].

This paper proposes multiple PVAs that compose of several shunt circuits to improve the robustness of the PVA. Multiplexing technique has been utilized for the mechanical vibration absorber as mentioned above. The proposed method is analogy of this. Optimizing the PVA is important, because the effect of the PVA strongly depends on its natural frequency and damping ratio. Then this paper firstly treats the optimization problem of multiple PVAs. After that, Numerical simulations and experiments are demonstrated to reveal the effectiveness and the robustness of the multiple PVAs.

2. Equations of motion of beam with piezoelectric patches

The equations of motion of a beam attached with several piezoelectric patches[8] shown in Fig. 1 are expressed as

$$\ddot{\boldsymbol{x}} + \boldsymbol{\Xi}\dot{\boldsymbol{x}} + \boldsymbol{\Omega}\boldsymbol{x} = \boldsymbol{B_1}\boldsymbol{q} + \boldsymbol{B_2}\boldsymbol{V_a}, \tag{1}$$

where $\boldsymbol{x}$ is the vector of the modal displacement, $\boldsymbol{q}$ is the vector of the electrical charges of piezoelectric ceramics, and $\boldsymbol{V_a}$ is the vector of voltage for disturbance. Constitutive equations are expressed as

$$\boldsymbol{V} = -\boldsymbol{B_p}^\top \boldsymbol{x} + \boldsymbol{C_p}^{-1}\boldsymbol{q}, \tag{2}$$

where $\boldsymbol{V}$ is the vector of voltages of piezoelectric ceramics. Definitions of above matrix are as follows:

$$\boldsymbol{M_b} = \mathrm{diag}\left[\rho_b A_b \int_0^{L_b} \Phi_k \Phi_k dx\right], \quad \boldsymbol{K_b} = \mathrm{diag}\left[E_b I_b \int_0^{L_b} \Phi_k'' \Phi_k'' dx\right],$$

$$\boldsymbol{C_p} = \mathrm{diag}[C_j], \quad \boldsymbol{L} = \mathrm{diag}[L_j], \quad \boldsymbol{R} = \mathrm{diag}[R_j]$$

$$\boldsymbol{M_p}[k,l] = \sum_{j=1}^{m} \rho_p A_p \int_{x_{j1}}^{x_{j2}} \Phi_k \Phi_l dx, \quad \boldsymbol{K_p}[k,l] = \sum_{j=1}^{m} E_p I_p \int_{x_{j1}}^{x_{j2}} \Phi_k'' \Phi_l'' dx,$$

$$\boldsymbol{B_p}[k,j] = -\frac{b_{pj}J_{pj}}{2w_p(x_{j2}-x_{j1})}\int_{x_{j1}}^{x_{j2}} \Phi_k'' dx, \quad \boldsymbol{B_e}[k] = \int_0^{L_b} \Phi_k dx,$$

$$\boldsymbol{M} = \boldsymbol{M_b} + \boldsymbol{M_p}, \quad \boldsymbol{K} = \boldsymbol{K_b} + \boldsymbol{K_p}, \quad \boldsymbol{\Omega} = \boldsymbol{M_p}^{-1}\boldsymbol{K_p}, \quad \Xi = \mathrm{diag}[\zeta_k], \quad \boldsymbol{B_1} = \boldsymbol{M_a}^{-1}\boldsymbol{B_p},$$

where $L_b, A_b, I_b, \rho_b, E_b$ are the length, the cross section, the geometrical moment of inertia, the density, and the Young's modulus of the beam. The jth patch is placed between x_{j1} and x_{j2}. Φ_k is kth modal function of a uniform clamped beam. $\boldsymbol{B_2}$ is the coefficient matrix of disturbances.

When the piezoelectric transducer is connected to a inductor and a resistor, Kirchhoff's laws of shunt circuit introduce

$$\boldsymbol{V} = -\boldsymbol{L}\ddot{\boldsymbol{q}} - \boldsymbol{R}\dot{\boldsymbol{q}}, \tag{3}$$

where L_j, R_j are the inductance and the capacitor of circuit connected with the jth PVA.

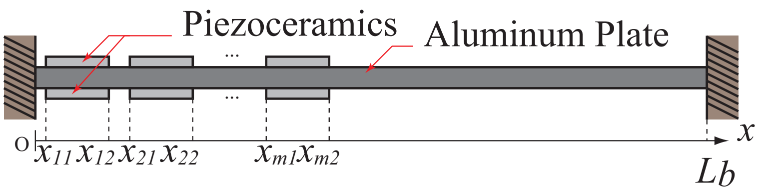


Figure 1. Analytical model of beam attached with piezoelectric patches

Suppose that the disturbance to the system is periodic, namely

$$\boldsymbol{V_a} = \tilde{\boldsymbol{V}}_{\boldsymbol{a}} \mathrm{e}^{\mathrm{j}\omega t}, \ \boldsymbol{x} = \boldsymbol{X}\mathrm{e}^{\mathrm{j}\omega t}, \ \boldsymbol{q} = \boldsymbol{Q}\mathrm{e}^{\mathrm{j}\omega t} \tag{4}$$

Then, we obtain

$$\boldsymbol{X} = \boldsymbol{G}(\omega)\tilde{\boldsymbol{V}}_{\boldsymbol{a}}, \tag{5}$$

where

$$\boldsymbol{G}(\omega) = \left[(\boldsymbol{\Omega} - \omega^2\boldsymbol{I}) + \mathrm{j}\omega\Xi - \boldsymbol{B_1}\{(\boldsymbol{\Omega_e} - \omega^2\boldsymbol{I}) + \mathrm{j}\omega\Xi_{\boldsymbol{e}}\}^{-1}\boldsymbol{L}^{-1}\boldsymbol{B_a}^{\top}\right]^{-1}\boldsymbol{B}_2 \tag{6}$$

and

$$\boldsymbol{\Omega_e} = \boldsymbol{L}^{-1}\boldsymbol{C}^{-1}, \quad \Xi_{\boldsymbol{e}} = \boldsymbol{L}^{-1}\boldsymbol{R}$$

The system expressed by Eq. (6) is a resonant system, and the shunt circuits behave as like the multiple dynamic vibration absorbers.

3. REAL-CODED GENETIC ALGORITHM

We utilize genetic algorithm (GA)[9] to solve robust optimization problems of multiple PVAs. GA is to be an effective method for finding the global optimum, because it does not require a unimodal distributions and differentiation potential of an objective function unlike a steepest descent method. However, the performance of the conventional binary-coded GAs that deal with design variables as bit strings consisting of '0' and '1' is not high when the design variables are real numbers, because it loses information about scales and variable dependency by the transform of the variables to the bit strings.

On the other hand, real-coded GAs (RCGA) utilize floating point representation. RCGA is able to produce offspring in the neighbourhood of parent individuals, because it directly treats real number vectors in crossover operation. In RCGA, the reproduction, the crossover and the survival selection are repeated until satisfying the termination condition. RCGA usually dose not need any mutation.

The unimodal normal distribution crossover (UNDX) is the crossover method tending to preserve the variable dependency [10] . UNDX-m[11,12] is the extended UNDX to utilize the information of multiple parents. The procedure is as follows.

1. Choose an arbitrary number $m+1$ of parents $\boldsymbol{x^1}, \boldsymbol{x^2}, \ldots, \boldsymbol{x^{m+1}}$ randomly from the population.
2. Calculate the center of mass of these parents be $\boldsymbol{p} = \sum_i \boldsymbol{x^i}/(m+1)$, and calculate the difference vectors $\boldsymbol{d^i} = \boldsymbol{x^i} - \boldsymbol{p}$.
3. Choose another parent $\boldsymbol{x^{m+2}}$ from the population randomly.
4. Calculate the length D of orthogonal elements of $\boldsymbol{d^{m+2}} = \boldsymbol{x^{m+2}} - \boldsymbol{p}$ to $\boldsymbol{d^1}, \boldsymbol{d^2}, \ldots, \boldsymbol{d^m}$
5. Let $\boldsymbol{e^1}, \boldsymbol{e^2}, \ldots, \boldsymbol{e^{N_d-m}}$ be the orthonormal basis of the subspace orthogonal to $\boldsymbol{d^1}, \boldsymbol{d^2}, \ldots, \boldsymbol{d^m}$.
6. Generate offspring $\boldsymbol{x^c}$ as Eq. (7).

$$\boldsymbol{x^c} = \boldsymbol{p} + \sum_{i=1}^{m} w_i \boldsymbol{d^i} + \sum_{i=1}^{N_d-m} v_i D \boldsymbol{e^i}, \tag{7}$$

where N_d is the number of design parameters. w_i and v_i are normal random numbers following $N(0, \sigma_\zeta^2)$ and $N(0, \sigma_\eta^2)$, respectively. Kita et al. [12] proposed

$$\sigma_\zeta^2 = 1/\sqrt{m}, \quad \sigma_\eta^2 = 0.35/\sqrt{N_d - m} \tag{8}$$

The procedure of the optimization in this paper is shown in Fig. 2. We utilize the number of the design parameters, N_d, as the number of parent individuals, m.

4. OPTIMIZATION

4.1. Nominal optimization

At first, we apply the proposal method to the nominal optimization problem to minimize of the H_∞ norm of the transfer function. The optimization problem of multiple PVAs is formulated as

$$\begin{aligned} \text{objective function :} &\quad \|\boldsymbol{G}(\omega)\boldsymbol{G}^\top(\omega)\|_\infty \\ \text{design parameters :} &\quad \omega_{e1}, \ldots \omega_{em}, \zeta_{e1}, \ldots, \zeta_{em} \\ \text{constraint :} &\quad \omega_{ej} > 0,\ \zeta_{ej} > 0, \quad j = 1, \ldots, m \end{aligned} \tag{9}$$

where

$$\omega_{ej} = \sqrt{\frac{1}{L_j C_j}}, \quad \zeta_{ej} = \frac{R_j}{2}\sqrt{\frac{C_j}{L_j}} \tag{10}$$

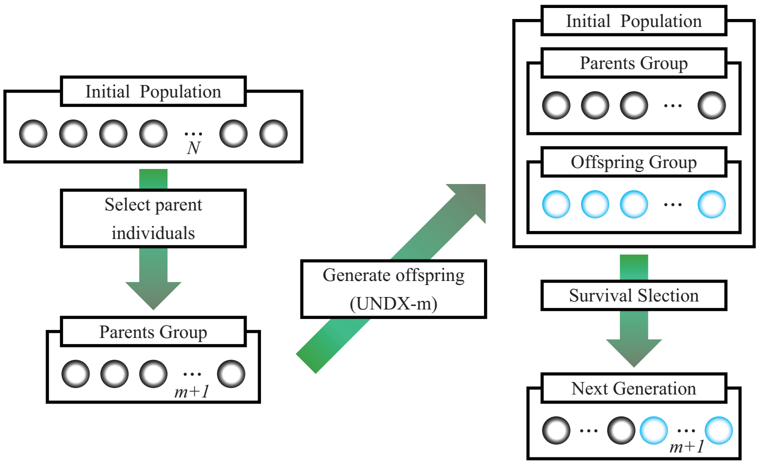


Figure 2. Procedure of RCGA

This paper deals with three type of PVAs, namely single ($m = 1$), double ($m = 2$), and quadruple ($m = 4$) shown in Fig. 3. Piezoelectric patches are attached on both sides. The patch pattern is the same as one of the experiment as stated below. Material constants and sizes are shown in Table 1, 2.

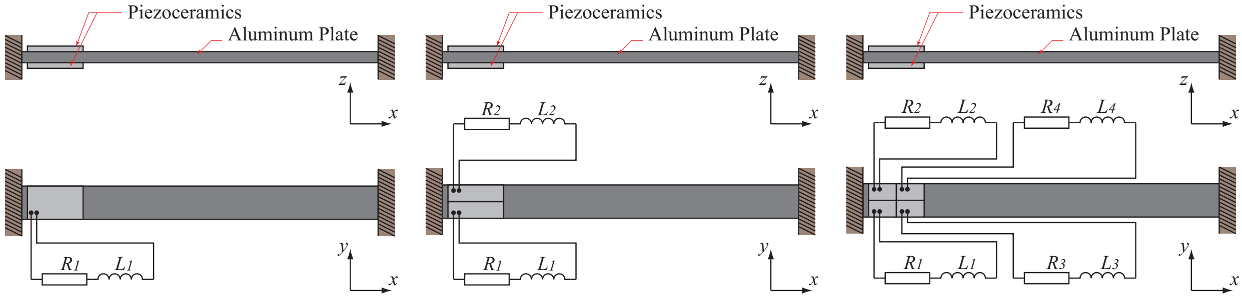
Table 1. Material constants for optimization

	Value	Unit		Value	Unit		Value	Unit
ρ_b	2700	kg/m^3	t_b	3.000	mm	ρ_p	8100	kg/m^3
E_b	71.00	GPa	A_b	60.00	mm^2	E_p	81.00	GPa
l_b	220.0	mm	I_b	45.00	mm^4	e_p	3.810	$\times 10^{-8}$ F/m
w_b	20.00	mm				b_p	-6.340	$\times 10^{8}$ N/C

Table 2. Sizes of piezoelectric patches for optimization (Unit: mm)

	l_p	w_p	t_p
single	40.00	20.00	0.5000
double	40.00	10.00	0.5000
quadruple	20.00	10.00	0.5000

The optimization result are shown in Table 3. Fig. 4 show the transfer functions of the system with single, double, and quadruple PVAs, respectively. The peak of transfer function of the beam with PVAs dramatically decreased compared with one of the beam without PVA, and the performance of quadruple PVAs was better than any other PVAs. This result reveal that the RCGA is useful for the optimal design of multiple PVAs.



(a) Single PVA (b) Double PVAs (c) Quadruple PVAs

Figure 3. Typical model of multiple PVA

Table 3. Optimal natural frequencies (rad/s) and damping ratios of PVAs

	single		double		quadruple	
i	ω_e	ζ_e	ω_e	ζ_e	ω_e	ζ_e
1	2347.3	0.0242	2374.7	0.0159	2320.1	0.0158
2	-	-	2319.2	0.0163	2376.3	0.0158
3	-	-	-	-	2410.3	0.0047
4	-	-	-	-	2284.3	0.0047
Maximum values	6.81		5.95		4.71	

4.2. Robust optimization

Model error considered is defined as

$$x_l = x + [-1 + 2(l-1)/m_r]\alpha x, \quad l = 1, 2, 3, \ldots, m_r + 1 \tag{11}$$

where x is a model parameter, α is a ratio of error, and m_r is the number of partitions to make an objective function. When the conditions: $\alpha = 0.05$, $m_r = 4$ are substituted, the transfer functions in Fig. 5 are obtained. Fig. 5(a) shows the transfer functions of the open system, and Fig. 5 (b) shows those of the system with the optimal quadruple PVAs of which natural frequencies and damping ratios are shown in Table 3. The effectiveness of damping at $l = 1, 2, 4, 5$ were far less than one at $l = 3$ that mean no model error.

In the robust optimization, given α and m_r, we propose to use the objective function as follows.

$$\begin{aligned} \text{objective function}: &\quad \max_{l} \left\{ \| \boldsymbol{G}_l(\omega) \boldsymbol{G}_l^\top(\omega) \|_\infty \right\} \\ \text{design parameters}: &\quad \omega_{e1}, \ldots \omega_{em}, \zeta_{e1}, \ldots, \zeta_{em} \\ \text{constraint}: &\quad \omega_{ej} > 0,\ \zeta_{ej} > 0, \quad j = 1, \ldots, m \end{aligned} \tag{12}$$

where $\boldsymbol{G}_l(\omega)$ is the transfer function in using x_l.

At first, the optimization problem was solved, when the error-contained model parameter x is the natural frequency of the main system. Fig. 6 (a) shows the comparison of the influence of the model error among the conventional single PVA and the robust optimized PVAs. These graphs are plots of maximum values of the transfer functions with respect to the natural frequency error.

The conventional single PVA well worked within range of 1.5 % of the error. However the effectiveness of the conventional PVA remarkably decreased beyond 1.5 % of the error. On the

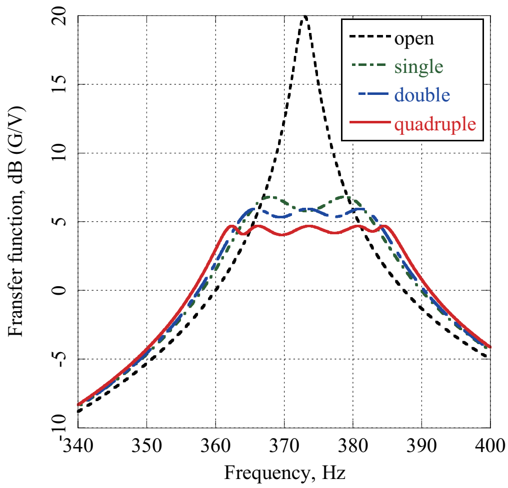
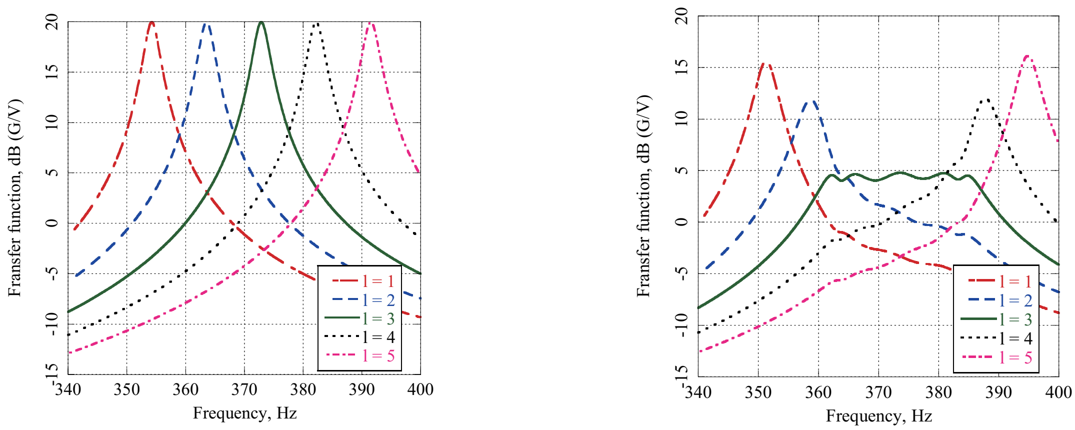


Figure 4. Comparison of transfer functions of the system with optimal-designed PVAs



(a) Open system (b) System with optimized quadruple PVAs

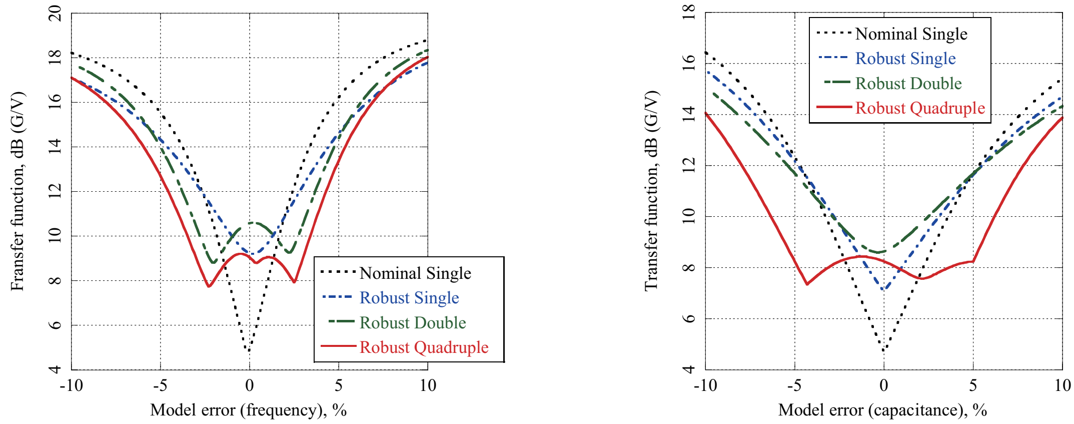
Figure 5. Influence of the natural frequency error of the main system to the transfer functions

other hand, the effectiveness of the robust optimized PVAs were more stable. If the requirement of damping is 10 dB, the conventional PVA allowed only ±1.6 %, but the robust quadruple PVAs allowed ±3.4 %. Then the maximum values of the transfer function with the conventional PVA was 14.7 dB, and much larger. This shows that the proposed method is useful for designing multiple PVAs to be robust.

Similarly, the optimization problem was solved, when x are the capacitances of the piezoelectric transducer. Fig. 6 (b) shows the multiple PVAs were robust than the conventional PVA as well.

4.3. Influence of patch pattern

The above section dealt with one patch pattern of piezoelectric transducers that is the same as one of the experiment. In this section, we describe the influence of patch pattern. We consider three pattern of the patches of the quadruple PVAs as in Fig. 7. The optimization problem is the

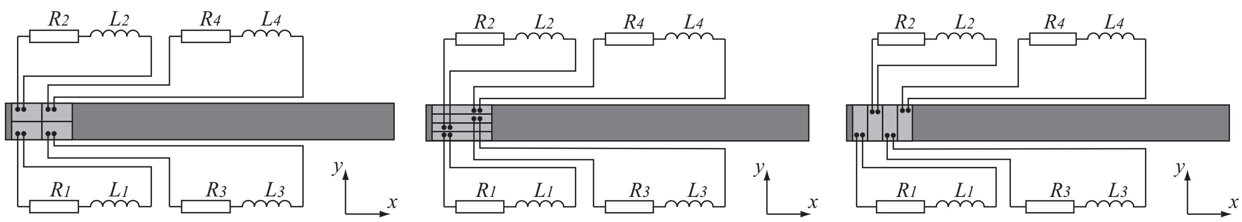


(a) Robustness to the natural frequency error of the main system

(b) Robustness to the capacitance error of piezoelectric transducers

Figure 6. Maximum values of the transfer functions

same as Eq. (12). The result in Fig. 8 shows that the lenghwise pattern is the best pattern for the robust design.



(a) crosswise pattern

(b) lengthwise pattern

(c) broadwise pattern

Figure 7. Patch patterns of quadruple PVAs

5. EXPERIMENT

Here, we demonstrated the experiment about the robustness over the temperature change.

5.1. Instruments

We built simulated LR circuit based on Antoniou synthetic inductor[13] for easy tuning. We used the digital potentiometer instead of analog potentiometer for register of LR circuit. Digital potentiometer can be controlled by electrical signal. Fig. 9 (a) shows Antoniou synthetic inductor. The equivalent circuit impedance of the synthetic inductor is the same as an inductor L_{eq}.

$$L_{eq} = \frac{R_1 R_3 C_1 R_4}{R_2} \tag{13}$$

Fig. 9 (b) shows a print circuit board of the simulated LR circuit. This circuit consists of four shunt circuits. Each sub-circuit has two variable resistors to realize the desired natural frequency

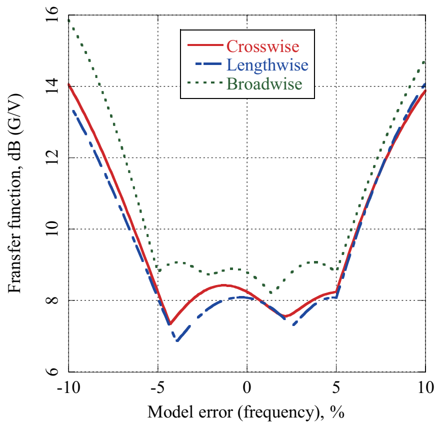
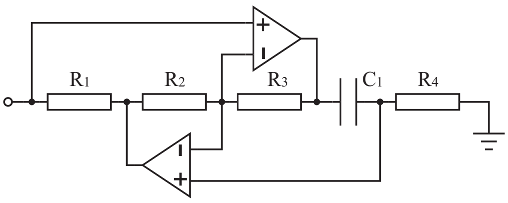
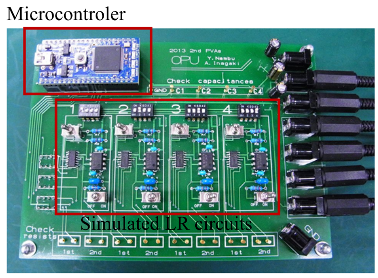


Figure 8. Comparison of robustness with the patch pattern



(a) Antoniou synthetic inductor



(b) PCB of Simulated LR circuits

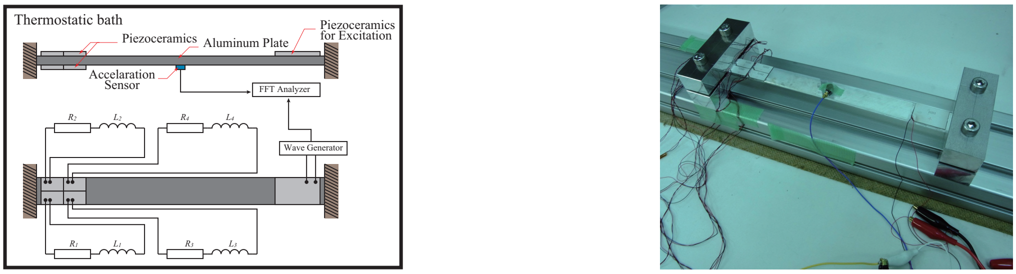
Figure 9. Simulated LR circuits

and damping ratio. The top toggle switches are for changing over from the series connection of the inductor and the resistor to the parallel connection. We only used the series connection in this study. And the dip switches are for changing over from the single PVA to the multiple PVAs. The bottom switches are for releasing piezoelectric transducers from the shunt circuit.

Fig. 10 shows the schematic figure and the photo of the experiment instrument. Clamped beam patched with piezoelectric ceramics in a thermostatic bath was the object of this experiment. Eight ceramics on the left side of the beam were utilized for multiple PVAs. They were patched on the front and the back of the beam. One ceramics on the right side was utilized for exciting the beam. This ceramics was connected to a wave generator. An acceleration sensor was placed on the center of the beam. Signal from the sensor and wave generator was inputted to FFT Analyzer.

5.2. Method

First, we optimized PVAs at 20 degrees Celsius. After optimization, changed the temperature from 20 degrees Celsius to 30, 40, 50, 10, 0, and -10 degrees Celsius and demonstrated four type of sweep tests at each temperature conditions, namely with the open circuit, with the single PVA, and with the double PVA, and the quadruple PVA.



(a) Schematic figure of experiment instrument (b) Photo of experiment instrument

Figure 10. Experiment instrument

5.3. Result

Fig. 11 (a)shows the transfer functions of the beam with no PVA, single PVA, and double PVAs, and quadruple PVAs under 20 degrees Celsius. The peak of transfer function of the beam with PVA dramatically decreased compared with one of the beam without PVA. Therefore multiple PVAs are useful to suppress vibration excited by the periodic disturbance. Fig. 11 also shows the performance of quadruple PVAs was better than any other PVAs.

Fig. 12 shows the maximum values of the transfer functions in several temperatures, namely, −10, 0, 10, 30, 40 and 50 degrees Celsius. The performances are all less than one in 20 degree Celsius. Fig. 12 also shows the robustness of quadruple PVAs is better than any other PVAs. The cause is the dependency of the capacitance of the piezoelectric transducer, which Fig 12 (b). Transfer functions of all temperature cases are shown in Fig. 13.

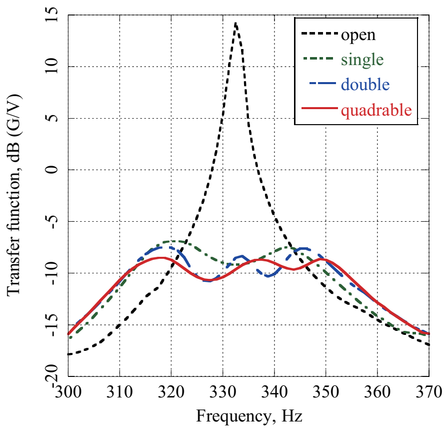
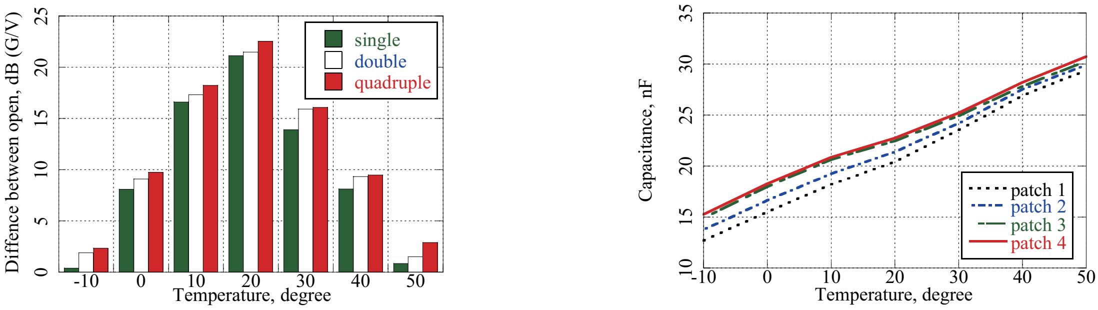


Figure 11. Frequency response of the beam with no PVA, single PVA, double PVAs, quadruple PVAs under 20 degrees Celsius.

6. CONCLUSION

This paper proposed multiple PVAs that compose of several shunt circuits in order to improve the robustness of the piezoelectric vibration absorber. The optimization problem to determine the



(a) Reduction vs temperature (b) Capacitance vs temperature

Figure 12. Influence of temperature

frequencies and damping ratios of multiple PVAs was solved by the real-coded genetic algorithm. Numerical simulations revealed that multiple PVAs designed by the proposed method had twice wider tolerance of the change of the natural frequency of main structure. Furthermore, experiments using the thermostatic bath were conducted. The experimental results showed that the multiple PVAs are more robust than the single PVA under variable temperature environment. In conclusion, multiple PVAs are effective and robust vibration control method for adaptive structures.

ACKNOWLEDGMENTS

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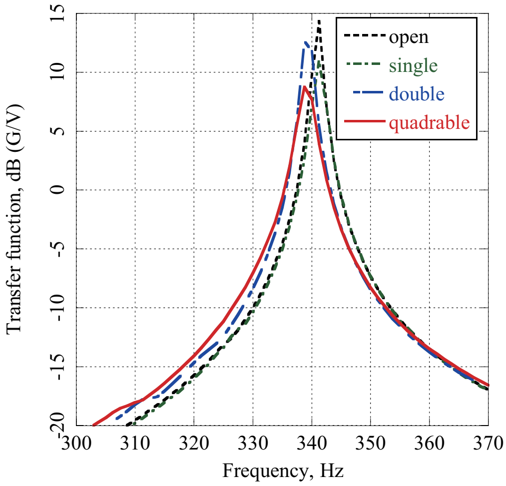
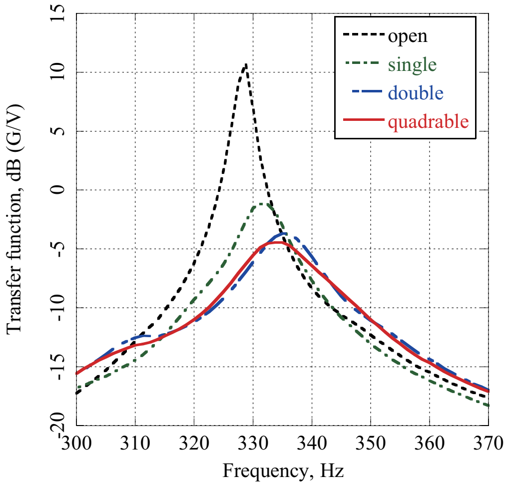
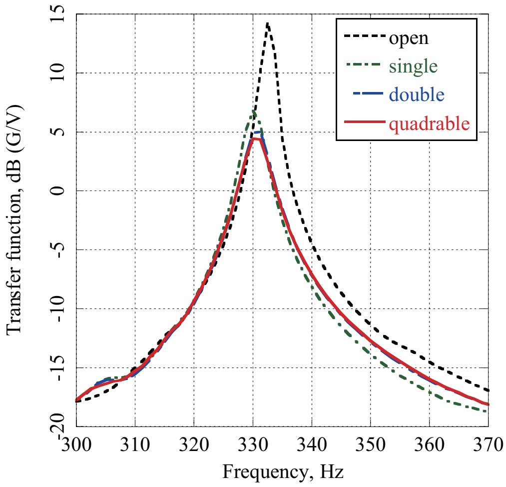


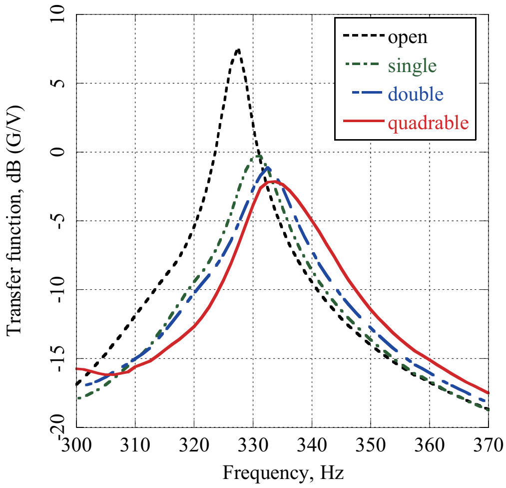
(a) -10 degrees Celsius



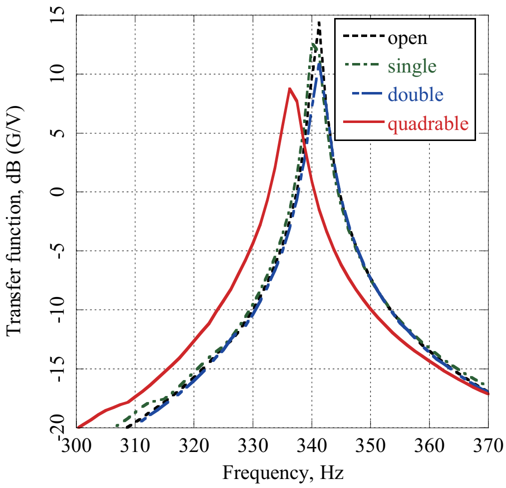
(d) 30 degrees Celsius



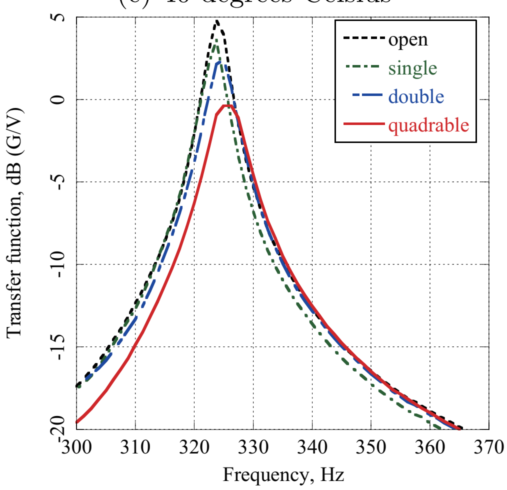
(b) 0 degrees Celsius



(e) 40 degrees Celsius



(c) 10 degrees Celsius



(f) 50 degrees Celsius

Figure 13. Frequency response of the beam with no PVA, single PVA, double PVAs, quadruple PVAs under several temperatures.