ICAST2014: 25th International Conference on Adaptive Structures and Technologies
October 6-8th, 2014, The Hague, The Netherlands

ICAST2014 #043

Adaptive Structural Shape Refinement by Means of SMA Wire (Verification of Basic Idea via Simulation Studies)

Kazuyuki Hanahara[1]* Yukio Tada[1]

[1] Department of Systems Science, Graduate School of System Informatics, Kobe University, Kobe, Japan

Abstract

Attaining a specified shape within a required accuracy is one of the basic missions for many structural systems. In order to meet this requirement, a kind of refinement or adjustment may be required for the structures; however, the maintenance by human engineers can be difficult or even impossible in some situations, such as in the case of an antenna structure of an on-orbit satellite. In this study, we deal with the improvement of structural accuracy by means of a shape refinement process with simple heating-and-cooling actuation of SMA (shape memory alloy) wires, which utilizes the intrinsic hysteretic characteristics of SMA. In order to cope with the inaccuracy of actuation results of individual SMA wires due to this simple actuation, we propose an adaptive multistep approach, where the SMA wires are actuated one single at a time. We deal with three types of algorithm to determine the actuating SMA wire in each step. Computer simulations are carried out with 2D truss structures; the basic feasibility of the proposed shape refinement by means of SMA wires with heating-and-cooling actuation is confirmed.

1. INTRODUCTION: BASIC IDEA

Attaining a specified shape within a required accuracy is one of the basic missions for many structural systems. In the case that a structure fails to satisfy the required accuracy of its shape for some reasons, adjustment of its structural parameters has to be carried out in order to achieve the intended shape requirement. Maintenance should be performed in general; however, it can be difficult or even impossible in some cases, such as in the case of an antenna structure of an on-orbit satellite. This is considered to be a typical situation that the adaptive structure technique[1] can be applied.

In the case of such geometrical or morphological adaptation, the selection of the actuation device and the manner of its implementation on the target structure are significant. For example, the conventional variable geometry truss (VGT), which is a typical structural system having geometry adaptation capability, adopts telescopic actuators as its truss members[2, 3] and has a statically determinate topology. The actuators and the implementation manner of this type enable the VGTs a finite shape change.

In the current study, on the other hand, we deal with a slight modification of structural shape in order to attain its specified geometrical requirement; this is a kind of shape refinement in other words. Such shape refinement is expected to be especially effective for a large-scale space structural system such as the parabola reflector for VLBI space observatory system; its required accuracy is 0.4mm rms for a 9m diameter structure in on-orbit condition[4, 5], for example. On the basis of the circumstances of

* Corresponding author; email: hanahara@cs.kobe-u.ac.jp

such structural system, the adaptive structure system in our study should be of maintenance free. Shape memory alloy (SMA)[6] wire is adopted as the actuator, since it is considered to be more reliable than typical mechanical actuators such as the electric motors having movable parts. In this study, SMA wires are implemented on the target structure in a statically indeterminate manner so that the resultant shape change caused by the phase transformation of the SMA wire is kept adequately small in accordance with the intended shape refinement.

The key feature of SMA taken up in this study is its hysteretic characteristic. On the basis of this hysteresis, the mechanical state of an SMA wire before and after the heating-and-cooling operation becomes inevitably different in general; the force due to the difference in its stress state results in the deformation of the target structure. It should be noted that the deformation is kept without further energy consumption for the heating in this case, once it has accomplished. We propose to take advantage of this conspicuous characteristic of SMA wire for the adaptive refinement of target structure shape.

The adaptive structure system dealt with in this study is assumed to have a considerable number of SMA wires in order to achieve the intended shape refinement, since the required shape modification can be of various pattern and the deformation attained by the actuation of any single SMA wire by means of the heating-and-cooling operation mentioned above is considered to be relatively insignificant as well as inaccurate. We discuss an actuation algorithm taking account of the situation; an adaptive multistep approach is proposed. In order to verify the basic idea of this shape refinement approach, simulation studies are conducted with truss structures.

2. STATIC BEHAVIOR OF STRUCTURAL SYSTEM WITH SMA WIRES

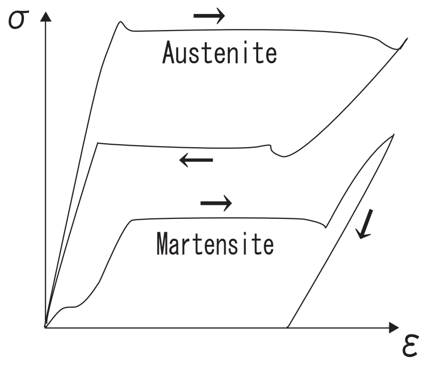
In order to determine the actuation pattern to meet the required shape deformation, as well as to examine the possibility of the proposed structural shape refinement approach by means of computational simulations, the static behavior of a structure having SMA wires has to be calculated. We do not take account of geometrical nonlinearity in the following, since the aim of the current study deals with only the infinitesimal deformation for the structural shape refinement. The detail of static behavior of structural system having SMA wires based on the piecewise linear characteristic model is dealt with in such as [7] and [8] for truss structure systems.

2.1 Piecewise-Linear Model of SMA Behavior

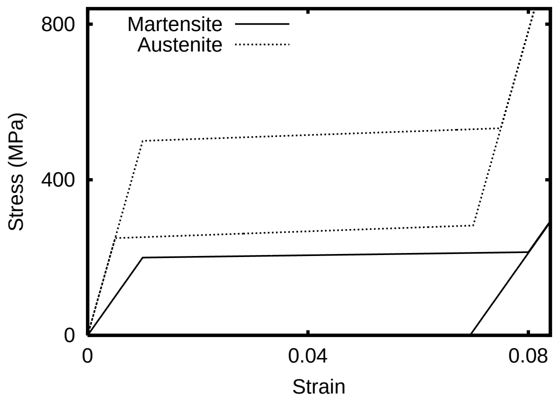
SMA has two crystalline phases called the martensite in low temperature condition and the austenite in high temperature condition. In both phases, its behavior exhibits the hysteretic characteristic and high nonlinearity. Figure 1(a) shows such a typical stress-strain relation of SMA. An accurate model of such SMA behavior is fairly elaborate[6] for computation of a system consisting of a number of SMA elements. In addition, our approach of structural shape refinement to be proposed does not assume accurate actuation of the individual SMA wires. We adopt a piecewise-linear model of the stress-strain relation of SMA shown in Fig.1(b) in this study.

The stress-strain relation of SMA is not one-to-one correspondence due to the hysteresis. We formulate the mechanical behavior of SMA based on the introduced piecewise-linear model in the following form:

$$E_+ = E_+(\epsilon, \sigma) \qquad E_- = E_-(\epsilon, \sigma) \tag{1}$$



(a) Typical SMA characteristics



(b) Piecewise-linear model

Figure 1. Piecewise-linear model of stress-strain relation of SMA

where ϵ and σ are the strain and stress of the SMA element and E_+ and E_- are the Young's moduli corresponding to the cases of increasing and decreasing strain. These Young's moduli take different values at the upper and lower boundary of the hysteretic part of the piecewise-linear stress-strain relation shown in Fig.1(b).

2.2 Formulation and Solution of Static Problem

We deal with the deformation of the target structure in terms of an adequate vector denoted by $\boldsymbol{U}$; its corresponding stiffness matrix is expressed as $\boldsymbol{K}$. We denote the thermomechanical state of SMA wires by $\mathcal{S}$ that consists of the stress, strain and temperature of all the SMA wires, which depend on their thermomechanical history as well. Taking account of an assumed corresponding external force $\boldsymbol{F}$, the static equilibrium condition of structural system having SMA wires at deformation $\boldsymbol{U}$ and SMA state $\mathcal{S}$ can be formulated in the following form:

$$\boldsymbol{K}(\boldsymbol{U}+\Delta\boldsymbol{U})+\hat{\boldsymbol{K}}(\mathcal{S},\mathcal{D})\Delta\boldsymbol{U}+\hat{\boldsymbol{F}}(\mathcal{S})=\boldsymbol{F}, \tag{2}$$

where $\hat{\boldsymbol{K}}$ is the stiffness contribution of the SMA wires corresponding to the further deformation $\Delta\boldsymbol{U}$ and $\hat{\boldsymbol{F}}$ is the internal force due to the stress of the SMA wires. It should be noted that $\hat{\boldsymbol{K}}$ depends not only on $\mathcal{S}$ but also on $\mathcal{D}$, which denotes the direction of further deformation of SMA wires, since the Young's modulus of each SMA wire differs in accordance with the further deformation direction as in Eq.(1).

We adopt the following iterative approach to solve the equilibrium equation (2), since the further SMA deformation direction $\mathcal{D}$ in the equation actually depends on $\Delta\boldsymbol{U}$.

1. Assume the parameter $\mathcal{D}$ as all the SMA wires are to be elongated.

2. Solve Eq.(2) based on the assumed parameter $\mathcal{D}$ as

$$\Delta\boldsymbol{U}=\left(\boldsymbol{K}+\hat{\boldsymbol{K}}(\mathcal{S},\mathcal{D})\right)^{-1}\left(\boldsymbol{F}-\hat{\boldsymbol{F}}(\mathcal{S})-\boldsymbol{K}\boldsymbol{U}\right). \tag{3}$$

3. In the case that the deformation direction of SMA wires corresponding to the calculated further deformation $\Delta\boldsymbol{U}$ and the assumed $\mathcal{D}$ are inconsistent, update $\mathcal{D}$ accordingly and repeat from **2**.

4. Update the deformation of the target structure as $\boldsymbol{U} \leftarrow \boldsymbol{U} + \Delta\boldsymbol{U}$. The thermomechanical state of SMA wires $\mathcal{S}$ should also be updated.

It should be again noted that $\boldsymbol{K}$ is constant since the assumed deformation $\boldsymbol{U}$ is infinitesimal; however, $\hat{\boldsymbol{K}}$ and $\hat{\boldsymbol{F}}$ significantly depend on the thermomechanical state of SMA wires.

3. ACTUATION APPROACH TAKING ACCOUNT OF INACCURACY

It is not necessarily impractical to conduct an accurate control of SMA behavior[9]; however, such an approach requires sensing and control devices for each of the SMA elements and does not assume to be applied to our case that significantly utilizes the hysteretic characteristics in order not to require any energy consumption in its steady state after actuation. There can also be individual differences among the SMA characteristics in accordance with their own individual thermomechanical history. We develop an actuation approach taking account of such inaccuracy of individual SMA wires.

3.1 Adaptive Multistep Shape Refinement

We introduce a binary-valued vector $\boldsymbol{B}$ that denotes the actuation conditions of SMA wires; the values 0 and 1 of each element of $\boldsymbol{B}$ respectively represent before and after the heating-and-cooling actuation of the corresponding SMA wire. It should be noted that the transition of the element of $\boldsymbol{B}$ is irreversible and restricted to 0 to 1; that is, the heating-and-cooling actuation can be performed only once and cannot be cancelled.

Taking account of the inaccuracy, we conduct a multistep actuation; the heating-and-cooling actuation of SMA wire is performed one single at a time. Transition of the structure deformation $\boldsymbol{U}$ and the SMA thermomechanical state $\mathcal{S}$ based on the heating-and-cooling actuation of a single SMA wire can be expressed in the following form as

$$\{\boldsymbol{U}, \mathcal{S}\} \leftarrow \mathcal{T}(\{\boldsymbol{U}, \mathcal{S}\}, n), \tag{4}$$

$$\boldsymbol{B} \leftarrow \boldsymbol{B} + \boldsymbol{B}_n, \tag{5}$$

where $\mathcal{T}$ describes the transition of deformation and SMA state based on the heating-and-cooling actuation of SMA wire n and $\boldsymbol{B}_n$ is the binary-valued vector having 1 only for the element corresponding to SMA wire n.

The deformed geometry of the target structure is expressed as $\boldsymbol{X} + \boldsymbol{U}$, where vector $\boldsymbol{X}$ expresses the initial geometry without shape refinement. In order to evaluate the refined shape of the target structure, we introduce the geometry evaluation vector expressed as

$$\boldsymbol{W} = \boldsymbol{A}(\boldsymbol{X} + \boldsymbol{U}), \tag{6}$$

where $\boldsymbol{A}$ is the binary coefficient matrix that is adopted to extract the part to be evaluated from the geometry of the structure deformed by means of the SMA actuation. The error function to be minimized is expressed as

$$E = \frac{1}{2}(\underline{\boldsymbol{W}} - \boldsymbol{W})^T(\underline{\boldsymbol{W}} - \boldsymbol{W}) \tag{7}$$

where $\underline{\boldsymbol{W}}$ is the reference geometry to be attained by means of the shape refinement.

We propose the following adaptive multistep refinement of the target structure shape that is intended to gradually minimize the error E:

1. Measurement of the current error vector $\boldsymbol{W}_{err} = \underline{\boldsymbol{W}} - \boldsymbol{W}$.

2. On the basis of $\boldsymbol{W}_{err}$, determine SMA wire n to be actuated in this step, which has not yet been actuated, that is, whose corresponding element of $\boldsymbol{B}$ is 0. In the case that an adequate next SMA wire to be actuated is not found, terminate the shape refinement process.

3. Perform the heating-and-cooling actuation of SMA wire n. Update $\boldsymbol{B}$ accordingly and continue from **1**.

It should be noted that the actuation sequence of SMA wires cannot be determined beforehand, since the inaccuracy of the simple heating-and-cooling actuation result has to be taken into consideration in each step.

3.2 Strategy to Determine Actuating SMA Wire

In the above-mentioned adaptive multistep shape refinement procedure, the key point is how to determine the actuating SMA wire n in **2**, based on the obtained measurement of $\boldsymbol{W}_{err}$. We introduce the following shape refinement coefficient corresponding to the heating-and-cooling actuation of single SMA wire n:

$$\Delta \boldsymbol{W}_n = \boldsymbol{W}_n - \tilde{\boldsymbol{W}} = \boldsymbol{A}(\boldsymbol{X} + \boldsymbol{U}_n) - \boldsymbol{A}(\boldsymbol{X} + \tilde{\boldsymbol{U}}) = \boldsymbol{A}(\boldsymbol{U}_n - \tilde{\boldsymbol{U}}), \tag{8}$$

where $\boldsymbol{U}_n$ is obtained by means of the calculated transition (4) based on the actuation of SMA wire n from the initial condition $\tilde{\boldsymbol{U}}$ and $\tilde{\mathcal{S}}$. This value is not accurately corresponding to the actually attained value due to the above-mentioned situation in our case; it can be used to determine the SMA wire to be actuated, however, because we adopt the multistep one-by-one approach taking account of such inaccuracy.

These shape refinement coefficients are considered to be applicable throughout the refinement process, since we confine our shape refinement to the case of infinitesimal deformation. This means that although the computational cost for the calculation of the coefficient is not insignificant due to the requirement of iterative calculation of the transition process (4), it is only necessary to calculate them once for all of the SMA wires beforehand.

In the current study, we deal with the following three types of algorithm to determine the actuating SMA wire that has not yet been actuated, based on these shape refinement coefficients.

3.2.1 Local Search

Solve the following problem:

$$\text{Find } n \text{ such that } |\boldsymbol{W}_{err} - \Delta \boldsymbol{W}_n| \to \min \tag{9}$$

and use the obtained SMA wire n for the actuation in each step. This is a typical local search algorithm. Terminate the process in the case that further shape refinement cannot be attained, that is, in the case of $|\boldsymbol{W}_{err} - \Delta \boldsymbol{W}_n| \geq |\boldsymbol{W}_{err}|$ for the obtained SMA wire n.

3.2.2 Simple Global Search

Solve the following problem:

$$\text{Find } \mathcal{A} \text{ such that } |\boldsymbol{W}_{err} - \Delta\boldsymbol{W}_{\mathcal{A}}| \to \min, \tag{10}$$

where $\Delta\boldsymbol{W}_{\mathcal{A}}$ is expressed in terms of set $\mathcal{A}$ of non-actuated SMA wires as

$$\Delta\boldsymbol{W}_{\mathcal{A}} = \sum_{i\in\mathcal{A}} \Delta\boldsymbol{W}_i.$$

Problem (10) is a typical combinatorial optimization based on the shape refinement coefficients; the genetic algorithm[10] is adopted in this study. We use $n \in \mathcal{A}$ such that

$$|\boldsymbol{W}_{err} - \Delta\boldsymbol{W}_n| \to \min$$

as the SMA wire to be actuated. This is a kind of global search algorithm; although the one-by-one actuation is adopted in order to take account of the inaccuracy. Terminate the process in the case that further shape refinement cannot be expected, that is, in the case of $|\boldsymbol{W}_{err} - \Delta\boldsymbol{W}_{\mathcal{A}}| \geq |\boldsymbol{W}_{err}|$ for the obtained solution of (10).

3.2.3 Constrained Global Search

It is expected that the simple global search previously introduced tends to result in the heating-and-cooling actuation of many number of SMA wires. Taking account of the intrinsic actuation inaccuracy in our case, however, a fine shape deformation obtained as a combination of deformations, each of which is of relatively significant magnitude, might cause a deterioration in the shape refinement result.

In order to avoid such deteriorated results due to combinatorial actuation, we introduce the following constraint for the SMA wire i to be included in the solution set $\mathcal{A}$ of problem (10) as

$$\boldsymbol{W}_{err} \cdot \Delta\boldsymbol{W}_i \geq 0 \quad \text{for} \quad i \in \mathcal{A}. \tag{11}$$

That is, only the shape refinement coefficients in the same direction as the target error vector in the step are taken into consideration.

How to solve the problem (10) and the condition to terminate the shape refinement process are the same as the above simple global search.

4. DEMONSTRATION IN TERMS OF TRUSS STRUCTURE SIMULATION

In order to demonstrate and confirm the feasibility of the proposed shape refinement approach, computer simulations of 2D truss structure having SMA wire members are conducted. All of the simulated trusses consist of square truss units of 1m $\times$ 1m. The ordinary non-SMA truss members are aluminum of 50mm^2 cross-section. The SMA wire members are typical NiTi of 0.1mm^2 cross-section, whose mechanical characteristics in both martensite and austenite phases are modeled as shown in Fig.1(b). The target reference shape to be attained by means of the refinement is straight-line arrangement of the left-most nodal positions of the truss. We assume the error in truss member lengths as the fundamental cause of the geometrical error and they are given as uniform random values of ± 0.3mm, except if otherwise stated. Initial strain of the SMA wire members is assumed to be 4%; in order to simulate the actuation inaccuracy of individual SMA wires, initial strain error is also assumed for each of the SMA wires and given as uniform random values of ± 1%.

4.1 Shape Refinement for Geometrical Error

Figures 2 and 3 are typical results of shape refinement simulations of truss structures of 2×30 units. The diagonal members of the truss units of the right row are the SMA wires used for the shape refinement. The target shape of the leftmost nodes and the corresponding attained geometry by the truss are indicated by the lines placed left side of the truss, with the error magnification factor of 250. All of the results shown in Figs.2(b)-(d) and Figs.3(b)-(d) demonstrate the significant reduction of error values. Figure 3(b) shows a typical result corresponding to a local optimal solution obtained in case based on the local search. As shown in Figs.2(c) and 3(c), the simple global search gives comparably good results; however, the number of actuated SMA wires is considerably many. As shown in Figs.2(d) and 3(d), comparably good results are obtained by means of the constrained global search as well; it should be noted that the number of actuated SMA wires are significantly reduced compared to the case of the simple global search.

4.2 Shape Recovery against External Force

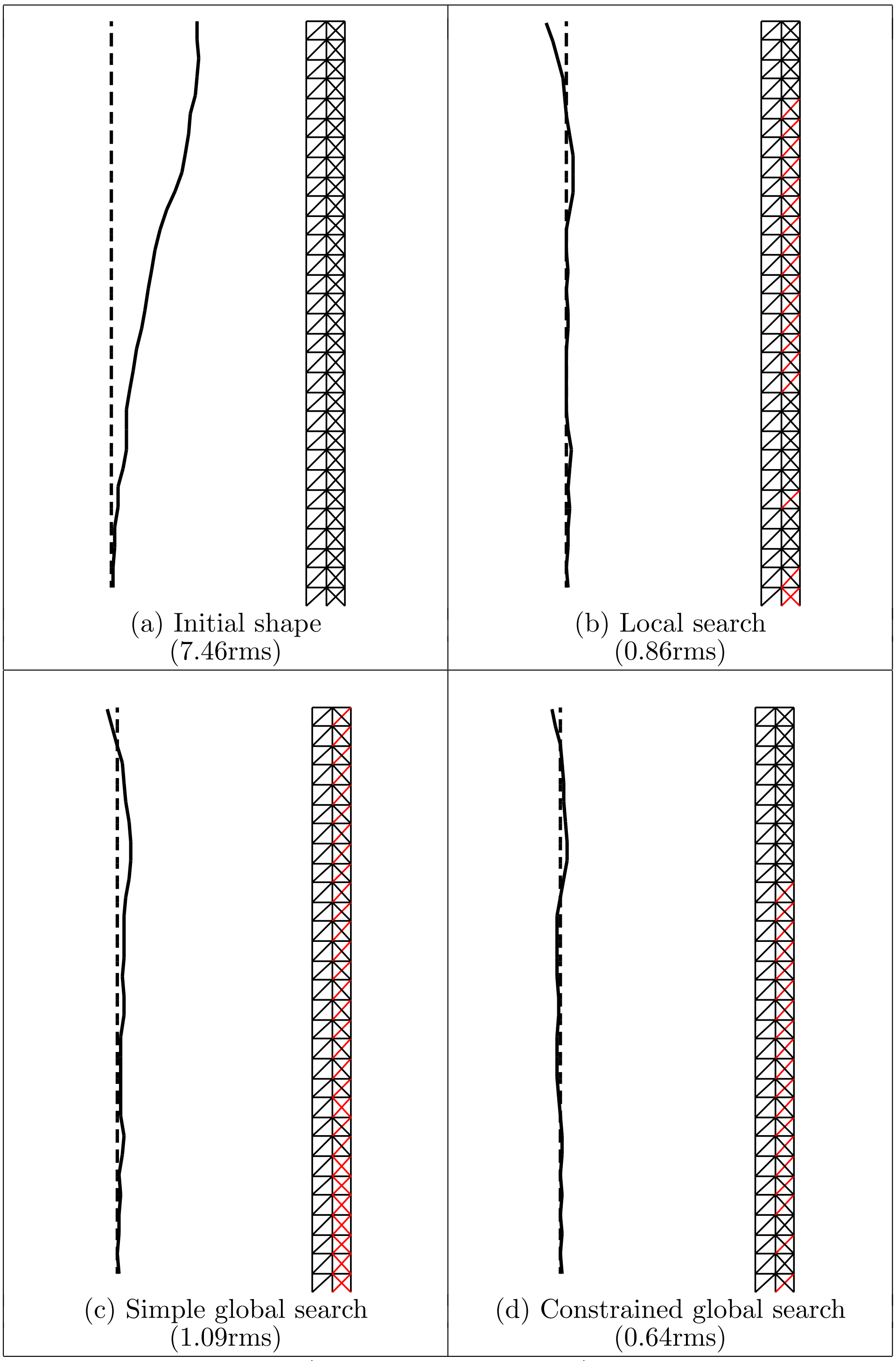
Figure 4 shows a typical simulated result of the shape recovery against an exerted horizontal external force of 5N at the top node. The error in truss member lengths in this case is given as uniform random values of ± 0.01mm; that is, the main cause of the geometrical error of the truss is the external force. Figure 4(a) shows the initial deformed shape due to the force and Fig.4(b) shows the result of shape refinement based on the constrained global search. The result demonstrates the feasibility of the shape recovery of this type based on the proposed SMA-wire approach.

5. CONCLUDING REMARKS

Hysteretic mechanical behavior is one of the intrinsic characteristics of shape memory alloys. In the current study, we utilize this characteristic of shape memory alloy and propose a shape refinement approach of structural system that does not require energy consumption for SMA actuation once the refinement has been completed.

The approach dealt with in this study is based on the implemented SMA wires on the target structural system. The simple heating-and-cooling operation is adopted for the actuation of individual SMA wires. In order to cope with the inaccuracy due to this type of actuation, we adopt an adaptive multistep shape refinement procedure, where SMA wires are actuated one single at a time. In order to determine the actuating SMA wire in each step, we deal with three types of algorithm in this study, that is, the local search, the simple global search and the constrained global search.

Computer simulation studies are carried out with 2D truss structures having SMA wires as a part of their truss members. On the basis of the obtained results, it is demonstrated that the RMS error to the reference target shape is significantly improved and reduced to $1/7 \sim 1/14$ by means of the proposed shape refinement. The approach based on the constrained global search gives comparably good results



(a) Initial shape (7.46rms)

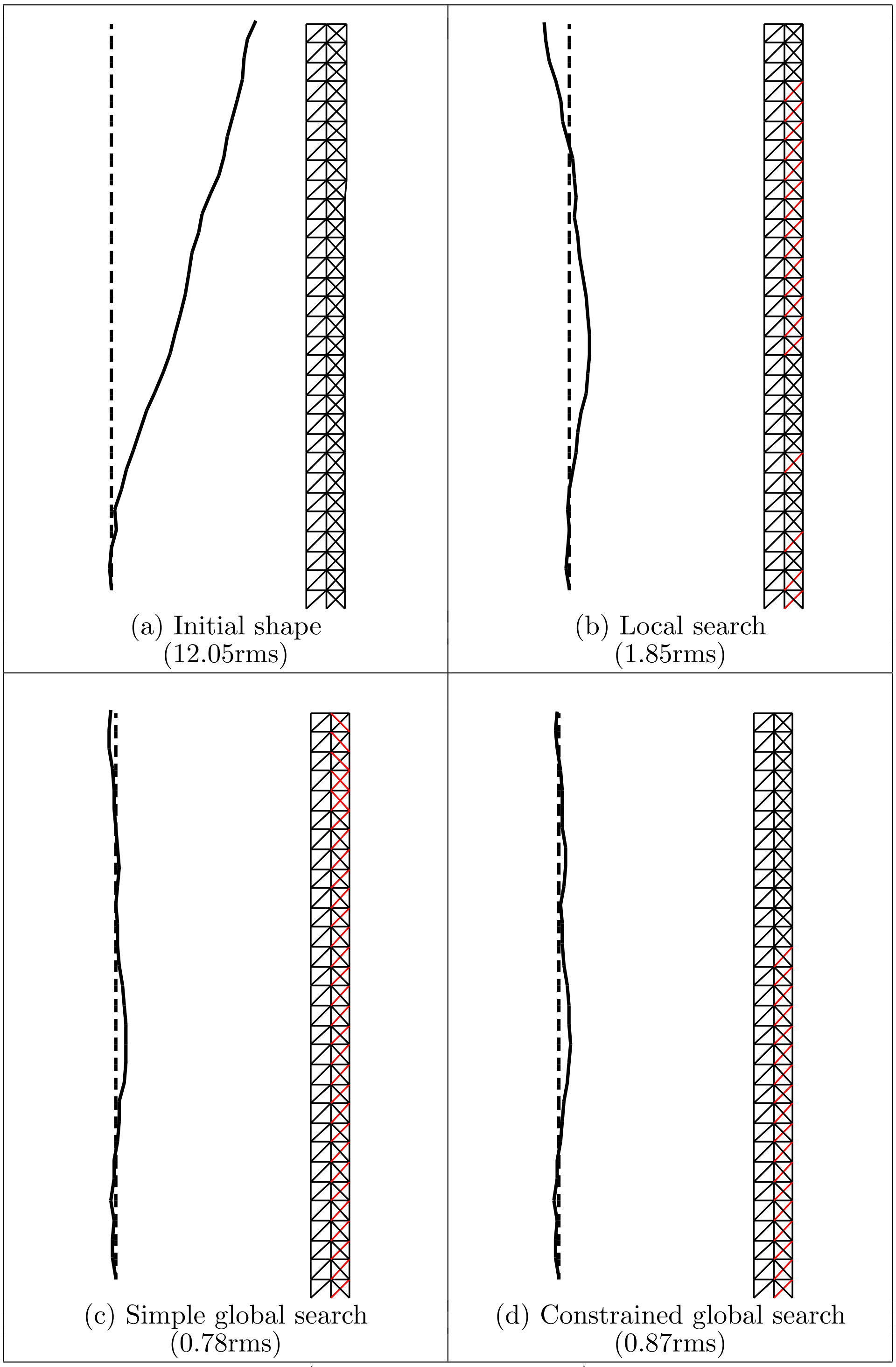
(b) Local search (0.86rms)

(c) Simple global search (1.09rms)

(d) Constrained global search (0.64rms)

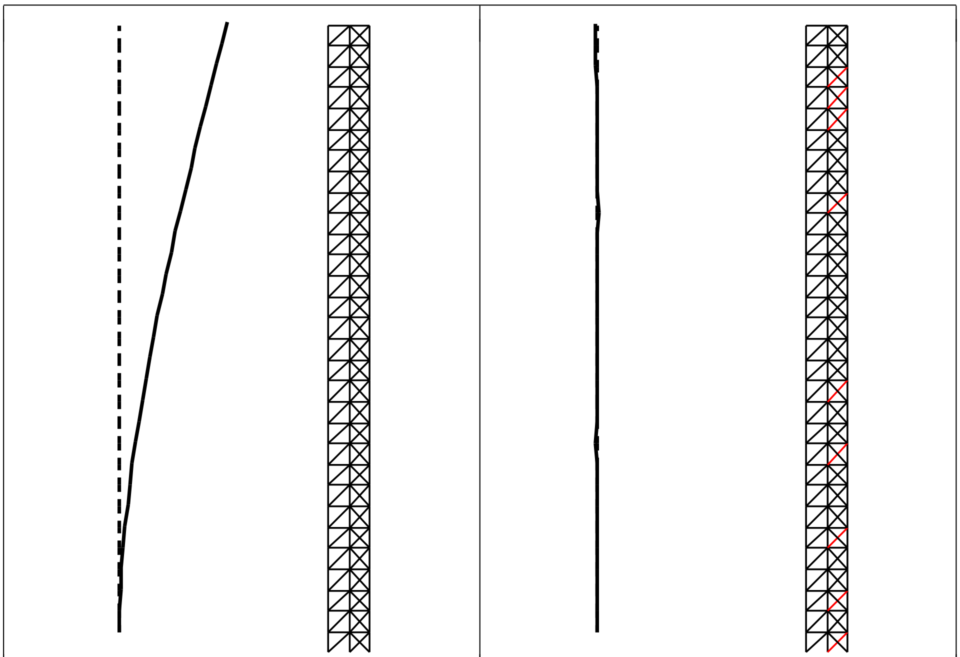
(Error magnified × 250)

Figure 2. Simulated results of shape refinement (1) (actuated SMA wire: −)
(Member length error: 0.3mm, SMA initial strain error: 1%)



(Error magnified × 250)

Figure 3. Smulated results of shape refinement (2) (actuated SMA wire: −)
(Member length error: 0.3mm, SMA initial strain error: 1%)



(a) Initial shape (7.40rms) (b) Constrained global search (0.02rms)

(Error magnified × 250)

Figure 4. Shape refinement on external force (actuated SMA wire: −)
(Member length error: 0.01mm, SMA initial strain error: 1%, Tip-end horizontal force 5N)

with relatively small number of actuated SMA wires. A shape recovery simulation result against an exerted external force is also conducted; the result demonstrates this feasibility as well.

It should be noted that the actuation of SMA wire by the heating-and-cooling operation can be performed without any sensing as well as control devices; it has been confirmed, however, that the proposed adaptive multistep actuation procedure can deal with the inaccurate situation considerably well. The shape refinement approach is considered to be applicable to many structural systems such as large scale antenna structures expected in the future space missions.

Practical designs including SMA-wire placement and its conditioning, application to other structural systems and experimental studies taking account of actual shape measurement instruments are the candidates of future works.

REFERENCES

1. e.g. Wada, B. K., Fason, J. L. and Crawley, E. F., “Adaptive Structures”, *Journal of Intelligent Material Systems and Structures*, Vol. 1, No. 2, 1990, pp. 157-174.
2. Miura, K., Furuya, H. and Suzuki, K., “Variable Geometry Truss and Its Application to Deployable Truss and Space Crane Arm”, *Acta Astronautica*, Vol.12, No.7/8, 1985, pp. 599-607.

3. Tanaka, M., Seguchi, Y. and Hanahara, K., “Kinematics of Adaptive Truss Permitting Nodal Offset (Configuration and Workspace Reach)”, *Journal of Intelligent Material Systems and Structures*, Vol.2, No.3, 1991, pp. 301-327.
4. e.g. VLBI Space Observatory Programme - 2, http://www.vsop.isas.jaxa.jp/vsop2/
5. Hirabayashi, H., “Next Generation Space VLBI”, *Radio Astronomy from Karl Jansky to Microjansky*, Gurvits, L. I., Frey, S. and Rawlings, S. eds., EAS Publications Series, Vol.15, 2005, pp. 465-478.
6. Lagoudas, D. C. eds., *Shape Memory Alloys (Modeling and Engineering Applications)*, Springer, 2008.
7. Hanahara, K. and Y. Tada, “Variable Geometry Truss with SMA Wire Actuators (Basic Consideration on Kinematical and Mechanical Characteristics)”, *Proceedings of the 19th International Conference on Adaptive Structures and Technologies*, 2008.
8. Hanahara, K. and Y. Tada, “Practicality Assessment of SMA-Wire Actuated Variable Geometry Truss”, *Proceedings of the 22nd International Conference on Adaptive Structures and Technologies*, 2011.
9. Cho, H., et al., “Exploitation of Shape Memory Alloy Actuator Using Resistance Feedback Control and Its Development”, *Progress in Natural Science: Materials International*, Vol.20, 2010, pp. 97-103.
10. e.g. Reeves, C. R. eds., *Modern Heuristic Techniques for Combinatorial Problems*, John Wiley & Sons, 1993, Chap. 4.