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Concurrent Design of Adaptively Damped Structures

Tommaso Delpero[1], Andrea Bergamini[1*], Paolo Ermanni[2]

[1] Laboratory for Mechanical Integrity of Energy Systems, Empa, Dübendorf, Switzerland
[2] Composite Materials and Adaptive Structures Laboratory, ETH, Zürich, Switzerland

Abstract

Previous work on the development of damping of methodologies using adaptive materials, such as piezoelectric elements, has demonstrated the effectiveness of smart damping in a great number of instances. In many instances, the main focus of the investigation was set on the identification of the optimal placement of adaptive damping devices applied to simple structures or to conventionally designed 'real world' structures, their physical integration into the structure, or design of effective laws as well as their implementation in analog circuits. In this contribution, we discuss a different level of integration of piezoelectric elements for damping purposes: the integration in the design process. In conventional (i.e. non-adaptive) structures, such as rotating machinery, vibration issues can be solved by increasing their stiffness, so as to shift the onset of resonant vibrations to higher frequencies. When in a second step piezoelectrics are applied, their authority on the structure may be limited. In the proposed concurrent design, the integration of piezoelectric elements is considered from the very beginning of the design of the structure. Here, the dynamic response of the structure is optimized by considering a trade-off between stiffness of the host structure and the ability of the piezoelectric elements to couple mechanical energy into the electric domain to achieve higher damping. The design space can thus be opened to less stiff and lighter solutions, which may be rejected by a conventional design approach, and a new set of lighter solutions becomes available with positive impacts on the outcome of the design process. In order to successfully carry out the concurrent design of piezo-damped structures, assumptions about the nature of the exciting forces need to be made. In this contribution, an example of the concurrent design of a structure with piezoelectric elements is given and considerations about different spectral distributions of the excitation are made.

1. INTRODUCTION

Adaptive materials have been intensively researched for the past few decades. Their promise is to introduce decisive elements of novelty in the design of mechanical systems through their ability to transduce energy from the mechanical domain to and from various others, such as the electrical, magnetic and thermal domains. Among the numerous explored adaptive materials, piezoelectric shunt damping has been proposed as a suitable method for controlling structural vibrations.

[*] Corresponding author, email: andrea.bergamini@empa.ch

The general method of shunt damping takes advantage of the high blocking stress, large bandwidth and relatively strong electromechanical coupling exhibited by piezoelectric ceramics, like PZT. As the piezoelectric element—bonded on or embedded in the structures—strains, a portion of the mechanical vibration energy is converted into electrical energy, which can be treated in a network of electrical elements connected to the piezoelectric patch. The technique was first introduced by Forward [1] to damp mechanical vibrations in optical systems, while Hagood and von Flotow [2] provided the first analytical formulation for the passive shunts.

Besides the development and characterization of piezoelectric materials, most of the recent and past work has focused on investigating new shunt circuits [2, 3, 4, 5], the structural integrity of the piezo-augmented structure [6, 7, 8], new finite element formulations [9, 10, 11] also including the numerical models of the controllers or the shunt circuits [12, 13], and the optimization of their size and placement [14, 15].

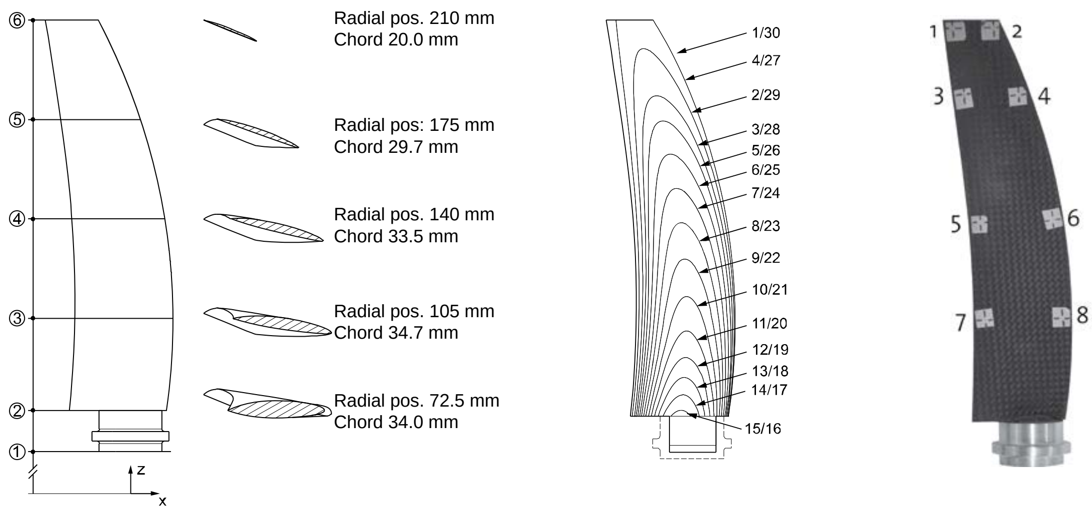
In this work, we discuss a different level of integration of piezoelectric elements for damping purposes: the integration in the design process. The analysis of the response of structures with shunted piezoelectric elements circuit [16, 17, 18] has shown that the amount of obtainable damping depends only on the electromechanical coupling of the piezo-augmented structure and some characteristics of the electrical circuit [19]. This implies that, for a given size of the piezoelectric element, a stiffer structure will result in a smaller portion of strain energy available in the piezoelectric material to be transformed in electrical energy, and consequentially in a smaller electromechanical coupling and lower damping. This consideration suggests the need for a compromise between the stiffness of the host structure and the ability of the piezoelectric elements to couple mechanical energy into the electric domain in order to satisfy both the static and dynamic requirements: While in conventional structures, the intuitive solution of a stiffness increase brings benefit to the mechanical system, there are cases—such as piezo-augmented structure—where a higher stiffness may lead to suboptimal solutions with respect to the dynamic requirements.

In this contribution, we present the concurrent design of a scaled model of a rotating blade, in which shunted piezoelectric elements are considered as additional design variables since the early design phases. It is shown that the additional degrees of freedom offered by the adaptive damping can open the design space to a new set of less stiff and lighter solutions, which would otherwise be rejected by a conventional design approach based on maximizing the stiffness. In order to accomplish the concurrent design, a method is suggested to compare passive and adaptive design solutions in terms of vibration amplitudes, independently from the nature of the damping source.

2. DESCRIPTION OF THE CASE STUDY

The selected case study is a scaled model of a of carbon fiber reinforced plastic (CFRP) propeller blade for wind tunnel testing, which has been investigated in the framework of a master thesis [20] carried out as collaboration between ETH Zurich and RUAG Aviation.

The investigated blade is meant for aerodynamic tests of counter-rotating open rotors, whose propellers are generally characterized by complex three-dimensional geometries with small cross sections, high stresses due to the centrifugal loads and are typically made out of CFRP. Counter-rotating open rotors are widely recognized as highly energy efficient propulsion systems. It is also well-known that their blades are subjected to large vibrations, mainly because of the rich spectrum of the aerodynamic forces. Since the purpose of the scaled model of the blade is to verify the aerodynamic performance of the propeller in a wind tunnel test, the structural vibrations have to be minimized in order to limit the spurious aerodynamic effect caused by its vibrations. The blade has to satisfy another important strength requirement: it has to carry the centrifugal load resulting from the nominal rotational speed of 10’000 RPM. The weight of the blade has a major impact on this second requirement, since the resulting centrifugal stresses are proportional to its mass. However, a lightweight solution, which reduces the centrifugal stresses at the root of the blade, typically has also the tendency to be subjected to large amplitude vibrations and may be not acceptable if not properly damped. The conflicting requirements, which constrain the design space of the blade, make this problem an appropriate example to show how the consideration of adaptive elements in the early design phases may lead to significant advantages in terms of performance of the whole structure.



(a) (b) (c)

Figure 1. Geometry of the investigated blade. (a) Aerodynamic profiles at different sections along the span. (b) Lay-up of the carbon fiber layers. (c) Picture of the manufactured blade. A reflecting tape is glued on the blade in eight positions in order for the laser vibrometer to measure its velocity [20]

The external geometry of the blade is given by aerodynamic considerations and it is defined by five NACA airfoil profiles, as shown in Figure 1. The inner structure of the blade is composed of 30 carbon fiber layers (T700SC + Araldit 5052), arranged in a symmetric lay-up: a fabric is used for the external layer in order to improve the robustness and the surface quality of the external surface of the blade; all the other layers are uni-directional (UD). The orientation angles of the five UD layers wrapped around the wedge (see Figure 2) are fixed by the geometry of the blade. The orientation of all the other layers can be changed in order to tune the mechanical properties of the structure. The wedge and the foot are made out of aluminum 7075-T6. The piezoelectric patch is a QP16n transducer produced by MIDÉ with a high performance PZT ceramic (CTS - 3195HD).

The orientations of the carbon fiber layers are the ‘passive design parameters’ that can be used to tune the stiffness and the eigenfrequencies of the structure. The ‘adaptive design parameters’ are the possibility to introduce a piezoelectric patch and its position. In this work, a design strategy is proposed to maximize the advantages obtainable with an adaptive solution with respect to the vibration response of the blade.

2.1 Finite Element Modeling and Experimental Validation

A finite element (FE) model of the blade is developed in order to evaluate its natural frequencies and the electromechanical coupling (and therefore the attainable damping) of the piezoelectric patch. The software used for realizing the FE model, solving it and evaluating the results is ANSYS Mechanical APDL. The blade is modeled with solid elements, so that each carbon fiber layer contains one element in the thickness direction. The mesh has therefore to comply with the lay-up and it is created with the help of the program ANSYS Composite PrePost. Figure 2 shows a detailed view of the FE model of the root of the blade: five layers extend beyond the blade structure and are wrapped around a wedge in order to fix the blade to the foot. Contacts between the carbon fiber layers, the wedge and the foot are modeled using ‘Targe170’ and ‘Conta174’ elements. Displacement boundary conditions are then applied to the external surfaces of the foot.

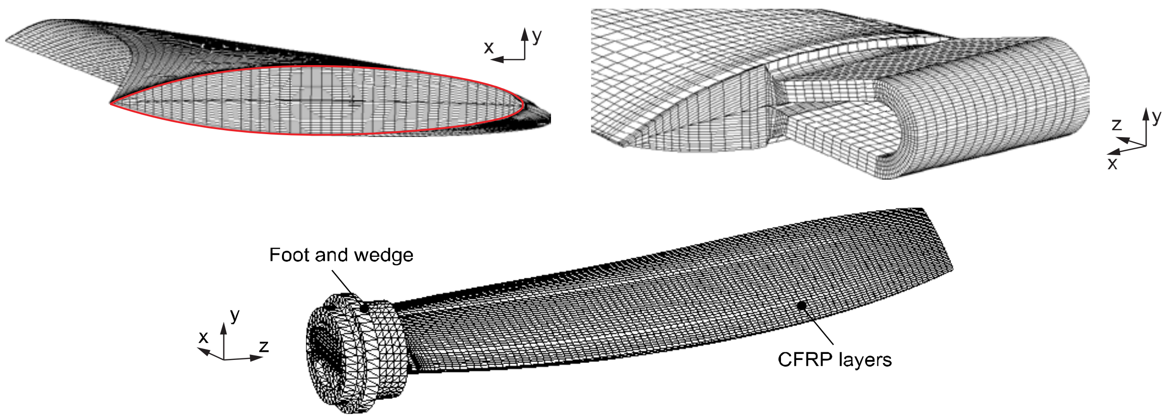


Figure 2 Finite Element model of the blade. Mesh and detailed view of the root of the blade **[20]**

The FE model has been experimentally validated with respect to its natural frequencies, the mode shapes and the electromechanical coupling of the piezoelectric patch. All the necessary measurements have been carried out with the experimental set-up shown in Figure 3. The blade is excited with a shaker using a stepped sweep signal. A load cell is used to measure the force input to the structure, and a laser vibrometer measures the velocity of the blade in the eight positions shown in Figure 1.

Two CFRP blades are tested. In both the structures, all the carbon fiber layers are oriented in the span direction (except for the ones wrapped around the wedge). One structure (blade #1) is used to validate the purely mechanical response of the FE model with respect to the natural frequencies and the shapes of the vibration modes. In the second structure (blade #2), a piezoelectric patch is embedded between the third and the fourth layer (see Figure 3) in a non-optimized position, with the only goal of validating the capability of the FE model to correctly represent the electromechanical coupling.

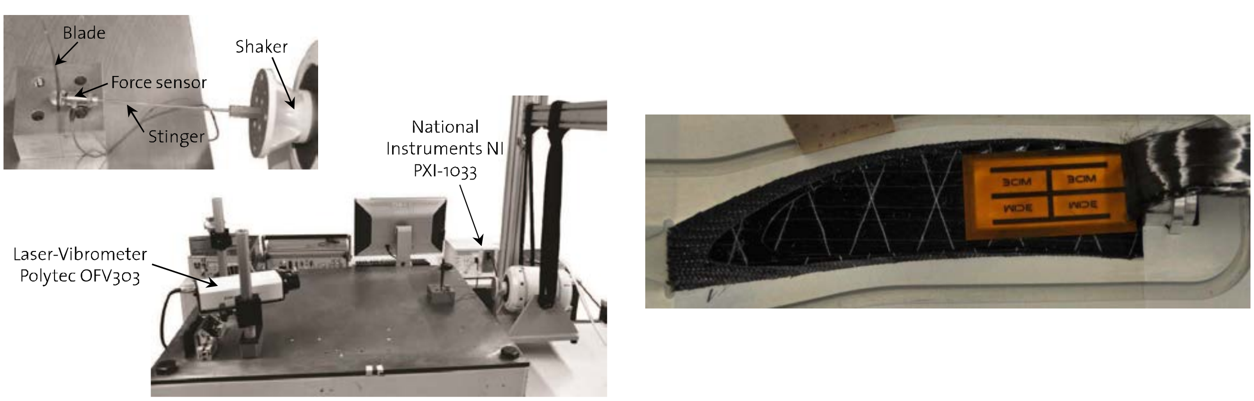


Figure 3. Picture of the test set-up and of the piezoelectric patch embedded in the blade during the lamination process

The amplitude and the phase of the corresponding Frequency Response Functions (FRF) are reported in Figure 4 for blade #1. The natural frequencies are compared with the ones obtained with the FE model in Table 1, showing a good agreement, with an error of approximately 5%.

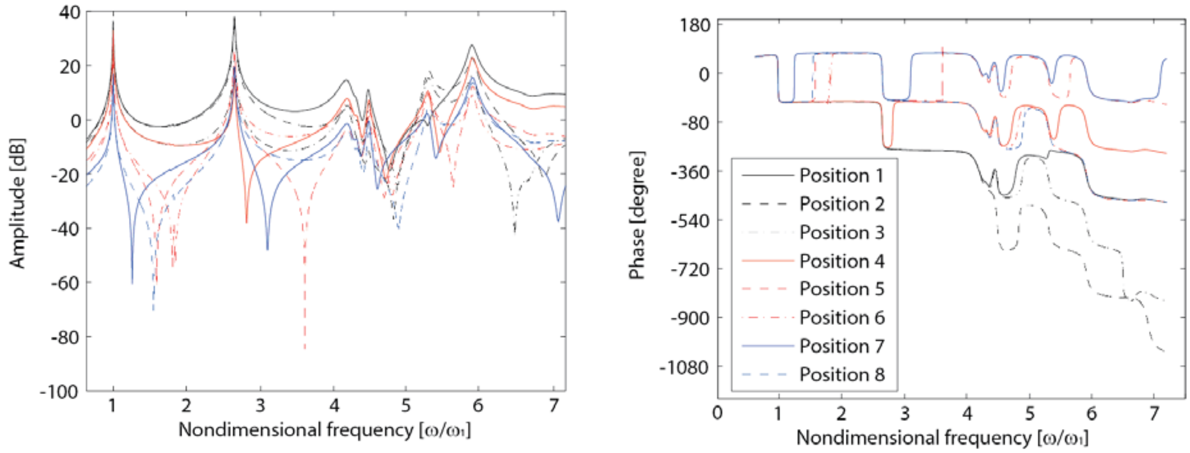


Figure 4. Amplitude and phase of the FRF functions measured for blade #1 **[20]**. The frequency is normalized with respect to the first eigenfrequency of the blade.

Table 1. Natural frequencies of blade #1. Comparison between finite element and experimental results.

	1st Mode	2nd Mode	3rd Mode	4th Mode	5th Mode
Difference FE - Exp	5.0 %	4.3 %	-4.7 %	-0.6 %	5.3 %

The capability of the FE model to correctly represent the electromechanical coupling is assessed in terms of the generalized Electro Mechanical Coupling Coefficient (EMCC). Numerically, it can be simply obtained from the natural frequencies of the blade when the piezoelectric patch is in open, ω_{OC} and short circuit, ω_{SC} $\left(EMCC = \frac{\omega_{OC}^2-\omega_{SC}^2}{\omega_{SC}^2}\right)$ [2]. The FE model of blade #2 predicts an EMCC for the first vibration mode of 4.3%.

Experimentally, the difference between the natural frequencies is too small to lead to an accurate measure, and a method based on application of the resonant shunt [19] is therefore applied. The FRFs obtained with three different electrical boundary conditions are reported in Figure 5. The dashed blue line corresponds to the piezoelectric patch in an open circuit state. The solid green line has been measured with the transducer connected to an optimally-tuned resonant shunt: a significant reduction in the vibration amplitude (10 dB) can be observed respect to the open circuit state, even if the position of the piezo has not been optimized. The solid red line is the FRF obtained when the piezo is shunted through a resonant shunt with a small value of resistance. The red line clearly shows the two peaks typical of a RL-shunt response and is used to evaluate the electromechanical coupling. The experimental value of the EMCC for the first vibration mode is 3.8%, which is in good agreement with the one predicted by the numerical model.

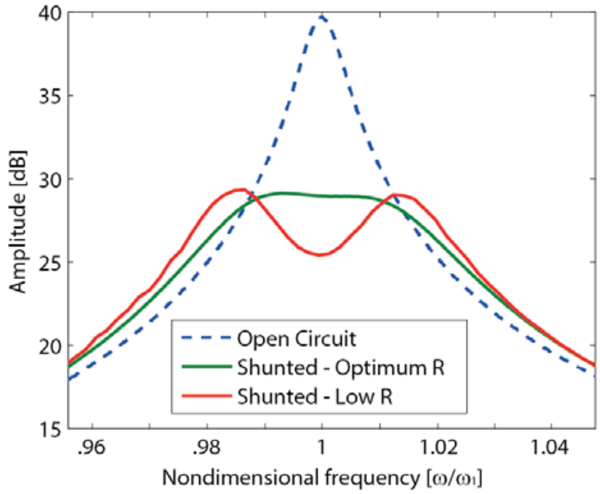


Figure 5. FRFs of blade #2 with different electrical boundary conditions for the piezoelectric patch **[20]**. The frequency is normalized with respect to the first eigenfrequency of the blade.

3. SEQUENTIAL AND CONCURRENT DESIGN APPROACHES

3.1 Definition of the performance indices

One of the main challenges related to a concurrent design is the definition of a common ground for comparing adaptive and traditional structures that, additionally, may feature any source of damping. The choice of a common performance index is not trivial, because of the large variety of structural properties (static stiffness, weight, natural frequencies and modal damping, for instance) that interacts in determining the final response of the system.

An elegant approach to the selection of the best material for minimizing the deformation of mechanical components caused by vibrational inputs has been proposed by Cebon and Ashby [21]. Different materials are compared using 'material selection charts' [22] combined with 'performance indices' (combinations of material properties). While their approach is strictly focused on materials, in this chapter it is applied to compare different structural designs, which may include both adaptive and purely passive solutions. The resulting performance indices do not merely consider the material properties, but the effective geometry of the structure and any other sources of damping are also included.

The problem of minimizing the vibration of a mechanical system is analyzed with respect to two load cases, which lead to two different types of performance index. In both the load cases, the input is a displacement excitation at the base of the structure. The performance indices proposed by Cebon and Ashby [21] are summarized in the following for the two considered load cases.

The first load case applies when the input is sinusoidal and its frequency is much smaller than the lowest natural frequency of the system. The frequency response function of the system can be approximated to the one of a single degree of freedom oscillator. For a given excitation frequency much smaller than the resonance frequency, the relative vibration amplitude is minimized when the natural frequency is maximized. Even in the case of realistic structures, with many eigenfrequencies and complex mode shapes, the general conclusion of maximizing the first natural frequency holds. Furthermore, the same conclusion of maximizing the stiffness of the system is reached if it is assumed that the input to the system is an oscillating force, rather than a displacement base input. Thus, the performance index (M_1) for the first load case coincides with the first natural frequency of the structure ($M_1 = \omega_1$).

The second load case applies when the excitation spectrum contains the natural frequencies of the system and most of the structural response occurs around resonant frequencies, where damping plays a key role. The broad-band input is assumed to be random with power spectral density $S_{xx} = S_0 \left(\frac{\omega}{\omega_0}\right)^{-k}$, where S_0, ω_0 and k are constants. When $k = 0$, the spectrum of the displacement input corresponds to a white noise input, which is generally unrealistic, because it implies an input with infinite power. When $k = 2$, the spectrum of the input velocity is constant and gives finite power. For $k > 2$ the input becomes more concentrated at low frequency. It is then possible to show [21] that, in the case of a broad band input, minimizing the root mean square of the relative vibration requires maximizing the performance index $M_2 = \eta_n \omega_n^{k-1}$, where η_n and ω_n are respectively the loss factor and the natural frequencies of the excited eigenmodes.

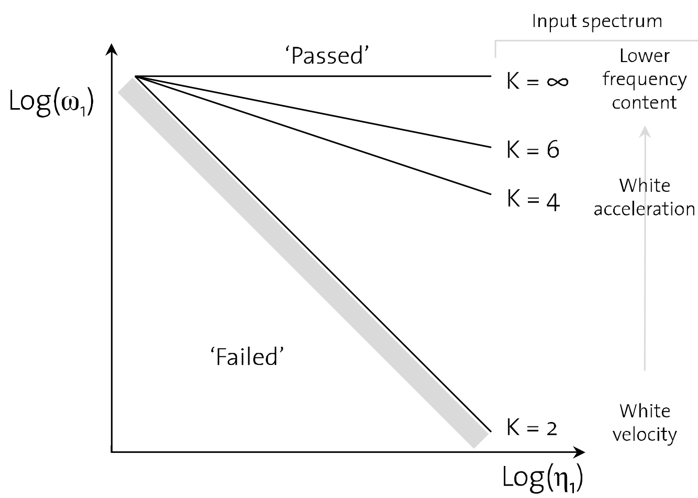


Figure 6. Schematic diagram for minimizing the root mean square deflection of a structure subjected to a random displacement input with spectral density $S_{xx} = S_0 \left(\frac{\omega}{\omega_0}\right)^{-k}$ [21]

If different design solutions are plotted in a diagram with $\log \eta$ on the x-axis and $\log \omega$ on the y-axis, all the solutions lying on the line of slope $1 / (1 - k)$ have the same performance index M_2. The concept is schematically shown in Figure 6. For k = 2, the slope of the selection line is -1. This means that, for a random input with a constant velocity spectrum, all the solutions below this line lead to larger deflections than the ones above the same line. For k → ∞ (input concentrated at low frequencies) the selection line becomes horizontal, and the selection task corresponds to maximizing the first natural frequency, as for the first sinusoidal load case. The diagram reported in Figure 6 clearly shows how the broader the input, the lower the efficacy of stiff structures. Indeed, when k is small (broad band input), a solution characterized by a lower natural frequency but higher damping may outperform a stiff one.

The diagram of Figure 6 does not make any assumption on the source of damping and can be therefore applied to the selected case study to compare the dynamic response of (i) a purely passive design with (ii) an adaptive structures designed with a sequential approach and (iii) an adaptive structures designed while considering the presence of piezoelectric elements since the early design phases.

3.2 Sequential Design

In a conventional design approach, only the purely mechanical design parameters (in this case study, they corresponds to the orientation of the carbon fiber layers) are initially considered, while the adaptive degrees of freedom (the position of the piezoelectric patch) are taken into account in a second phase. For this reason we will refer to this approach as ‘sequential design’, in contrast with the ‘concurrent design’ where both the passive and adaptive design parameters are concurrently considered in the early design phases.

Considering the nominal rotational speed of 10’000 RPM, the fundamental frequency of the excitation spectrum (166.7 Hz) is smaller than the lowest natural frequency of the system. If we initially assume that this is the only sinusoidal input, the deflection of the blade is minimized when the first eigenfrequency is maximized, as explained at the beginning of this section. A parameter study is conducted on the natural frequencies of the blade with respect to the orientation angles of the second and the third carbon fiber layers. Because the first modes are dominated by a bending behavior, this first design phase intuitively leads to a blade with all the layers oriented in the span direction, as shown in Figure 7.

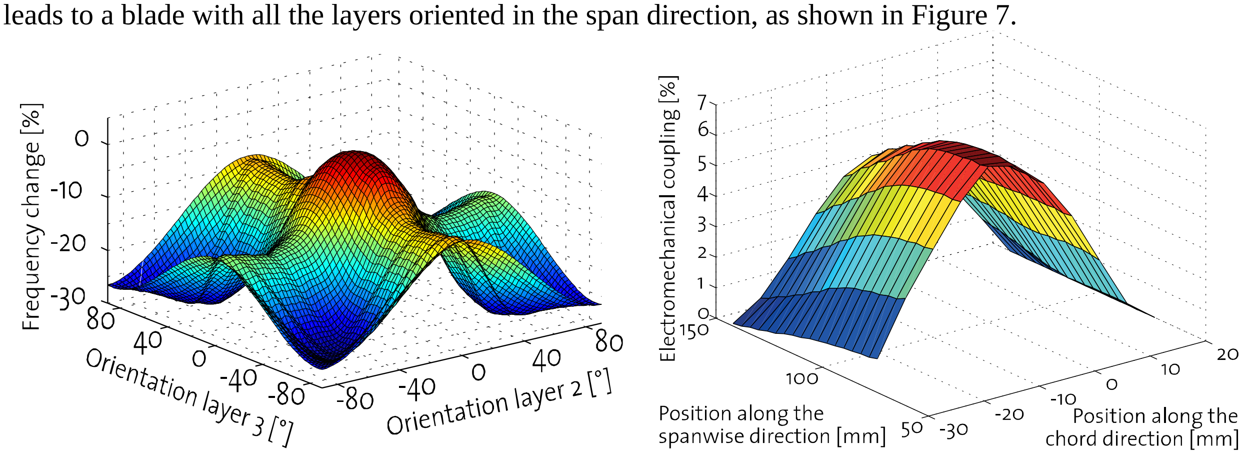


Figure 7. Parametric study: effect of (a) the orientation angles of the 2nd and 3rd layers on the first eigenfrequency and (b) the position of the piezoelectric patch on the generalized electromechanical coupling coefficient for the first eigenmode.

However, due to the complex aerodynamic interferences that characterize the dynamic of a rotating blade, additional damping may be considered in a second design phase in order to reduce the vibrations induced by higher-order harmonics. In this design phase, the lay-up is frozen with all the layers in the span direction, and the position of the piezoelectric patch is optimized in order to maximize the electromechanical coupling, and consequently the attainable damping. The design variables are the piezoelectric positions in the chord and in the span directions. The results of this parametric study are presented in Figure 7 for the first eigenmode. The optimal position lies close to the root of the blade. It results in an electromechanical coupling of EMCC = 6.4% that, assuming to connect the piezo to a tuned resonant shunt [19], would add a η = 4.5% of damping to the first vibration mode. On the other hand, the presence of the piezoelectric patch results in a small reduction of the first natural frequency (about 1%), due to higher density of the piezoelectric material respect to the carbon fiber layers.

Similar results are obtained for the second vibration mode. The optimal position lies about half the span of the blade, it results in electromechanical coupling of EMCC = 8.2% (that corresponds to an additional damping of η = 5.8%, if the piezo is connected to a resonant shunt) and a 4% reduction of the second natural frequency.

3.3 Concurrent Design

While a stiff solution minimizes the vibrations of a structure subjected to a low-frequency sinusoidal input, it may not be the optimal one when the excitation spectrum is broad-band and contains the natural frequencies of the structure. In this case study, the concurrent design consists of a simultaneous design of the position of the piezoelectric patch and the orientation of the carbon fiber layers. The ultimate goal is to find a compromise between the stiffness of the structure and the attainable piezoelectric damping.

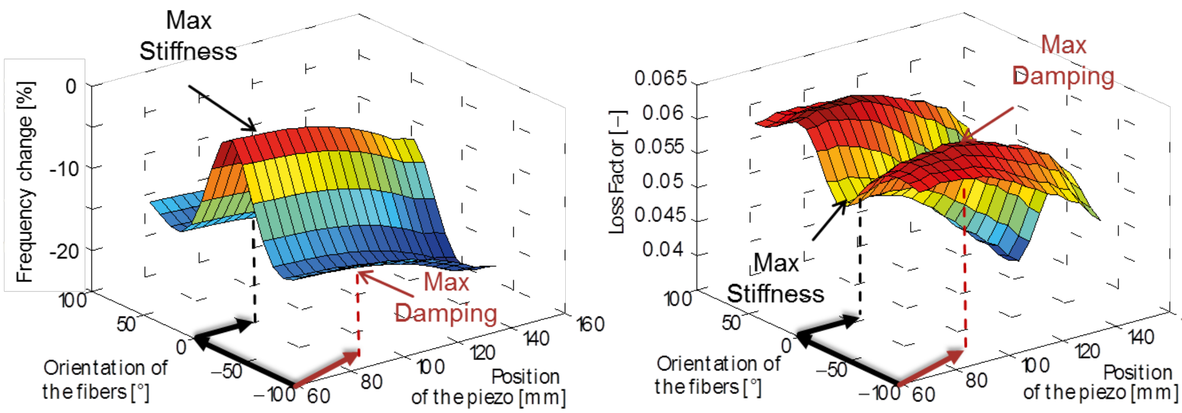


Figure 8. Results of the concurrent design of the blade in terms of loss factor and eigenfrequency for the first vibration mode. Solutions with low values of eigenfrequency lead to large values of loss factor.

The design variables selected for this example are the orientations of the second layer and the position of the piezo along the span direction. The results of the numerical simulations are depicted in Figure 8, where the first natural frequency and the attainable loss factor† are reported for each solution. The results clearly show the inverse relationship between the stiffness and the attainable damping: by simply accepting less stiff solutions (i.e. when the layers are not oriented in the span direction), the loss factor can be significantly increased.

Among all the possible solutions, the identification of the optimal one is not trivial, since it is neither the stiffest nor the most damped. As explained at the beginning of this section, the root mean squared of the deflection can be minimized by maximizing the product $\eta_n \omega_n^{k-1}$ of the vibration modes excited by the input. The design solutions are thus compared in Figure 9 on a single diagram, in terms of their first natural frequencies and loss factor.

The design solutions represented in Figure 9 can be assessed with respect to the performance index M_2, previously introduced. The black selection line corresponds to solutions that have the same performance index $M_2 = \eta\omega$, i.e. the same vibration amplitude when the input has constant velocity spectrum (k = 2). The red circle is the purely passive stiff blade resulting from the first design phase of the sequential approach. It has the highest eigenfrequency, and it is therefore the optimal solution only when the input is sinusoidal and its frequency is much smaller than the eigenfrequency (the selecting line in this case would be a horizontal line). The solution marked with the green circle is the result of the second phase of the sequential design. It has a slightly lower frequency than the purely passive one, but it has a much larger

† The loss factor depends on the electromechanical coupling and the shunt type. In this example, it is calculated assuming that the piezoelectric patch is connected to a resonant shunt. If the piezoelectric patch is connected to a different type of electrical shunt, other relations exist between the electromechanical coupling and the resulting damping [19]. The total damping of the structure can be calculated as the sum of the purely passive damping (measured during the experimental tests) and the additional piezoelectric shunt damping.

damping. However, this solution is not the optimal one when the input has a constant velocity spectrum. Among the solutions obtained with the concurrent approach (blue circles), the solution marked with the black star features the largest performance index $M_2 = \eta\omega$ and lead to the smallest amplitude deflections. It is interesting to observe that neither the solution with the largest damping is the optimal one, but a compromise has therefore been found.

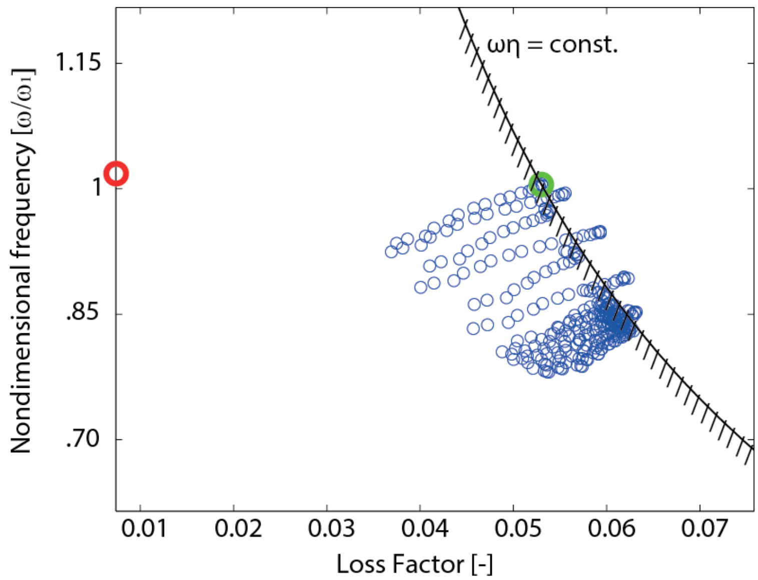


Figure 9. Results of the concurrent design of the blade. The loss factors and eigenfrequencies are reported on the same diagram in order to compare the solutions with respect to the performance index M2.

The choice of the best solution depends therefore on the spectrum of the input. When the excitation is concentrated at low frequencies (large values of k), the contribution of the eigenfrequency to the performance index $M_2 = \eta\omega^{k-1}$ is more important. On the other hand, when the excitation is broad band (small values of k) and contains the eigenmodes of the structures, the loss factor gains relatively higher importance, and less-stiff but highly damped solutions may be the best choice.

Additionally, if we now admit that the optimal solution, from a vibration point of view, is not the stiffest one, the stiffness reduction can be obtained by substituting carbon layers with a light core material, rather than changing their orientations. All the solutions lying on the selection line reported in Figure 9 are equivalent to the one obtained with a conventional approach in terms of vibration amplitude, but may be obtained with a lighter lay-up. The main advantage of the concurrent approach is therefore the possibility to extend the design space to lighter solutions that would otherwise be rejected because characterized by lower natural frequencies. The weight reduction will subsequently lead to a reduction of the centrifugal loads and enhancement of the strength of the whole blade, with clear beneficial impacts on the outcome of the design process.

4. CONCLUSIONS

The main contribution of this work is the suggestion of a common performance index and a method to design adaptive structures by minimizing the vibration amplitudes, independently from the nature of the damping source. The proposed method focuses on finding a compromise between the stiffness of the structure and the attainable damping.

The method has been applied to the realistic case study of the design of a rotating blade. This example shows how the adaptive solution outperforms the passive one (in terms of vibration amplitude) when the excitation spectrum is broad band or contains the eigenmodes of the structure. An accurate knowledge of the excitation spectrum is required for a quantitative comparison of the structural responses. On the other hand, when this knowledge is not available or uncertain, the adaptive design is in any case the most robust.

Additionally, the concurrent consideration of adaptive and passive design variables in the early design phases allows finding a better compromise between stiffness and damping than a conventional sequential approach. A concurrent approach that relies on a robust source of damping can extend the design space to less stiff solutions, which may be better or equally performing in terms of vibration amplitudes thanks to the adaptive element, and a new set of lighter solutions becomes available with positive impacts on the outcome of the design process.

The idea of relaxing the stiffness requirements of the structure may look counterintuitive when designing a structure with the ultimate goal of reducing its deformation under dynamic excitation. The method presented in this thesis allows quantifying when this approach is valuable. In particular, the proposed concurrent design approach is expected to be beneficial in highly constrained design problems, where the additional degrees of freedom offered by the piezoelectric material can help in relaxing the constraints on the natural frequencies of the structure.

The results presented in this contribution are part of the PhD thesis of Tommaso Delpero [23].

REFERENCES

[1] R. L. Forward, "Electronic damping of vibrations in optical structures," *Appl. Opt.*, vol. 18, no. 5, pp. 690-697, Mar 1979.

[2] N. W. Hagood and A. von Flotow, "Damping of structural vibrations with piezoelectric materials and passive electrical networks," *Journal of Sound and Vibration*, vol. 146, no. 2, pp. 243-268, 1991.

[3] L. R. Corr and W. W. Clark, "Comparison of low-frequency piezoelectric switching shunt techniques for structural damping," *Smart Materials and Structures*, vol. 11, no. 3, p. 370, 2002.

[4] L. R. Corr and W. W. Clark, "A Novel Semi-Active Multi-Modal Vibration Control Law for a Piezoceramic Actuator," *Journal of Vibration and Acoustics*, vol. 125, no. 2, pp. 214-222, 2003.

[5] S. Behrens, A. J. Fleming and S. O. R. Moheimani, "A broadband controller for shunt piezoelectric damping of structural vibration," *Smart Materials and Structures*, vol. 12, no. 1, p. 18, 2003.

[6] D. A. Singh and A. J. Vizzini, "Structural integrity of composite laminates with interlaced actuators," *Smart Materials and Structures*, vol. 3, no. 1, p. 71, 1994.

[7] J. P. Hansen and A. J. Vizzini, "Fatigue Response of a Host Structure with Interlaced Embedded Devices," *Journal of Intelligent Material Systems and Structures*, vol. 11, no. 11, pp. 902-909, 2000.

[8] M. M. Melnykowycz, "Long term reliabilty of Active Fiber Composites (AFC)," PhD thesis Nr. 17767, ETH Zurich, 2008.

[9] J. Becker, O. Fein, M. Maess and L. Gaul, "Finite element-based analysis of shunted piezoelectric structures for vibration damping," *Computers and Structures*, vol. 84, no. 31-32, pp. 2340-2350, 2006.

[10] O. Thomas, J.-F. Deü and J. Ducarne, "Vibrations of an elastic structure with shunted piezoelectric patches: efficient finite element formulation and electromechanical coupling coefficients," *International Journal for Numerical Methods in Engineering*, vol. 80, no. 2, pp. 235-268, 2009.

[11] F. Côté, P. Masson, N. Mrad and V. Cotoni, "Dynamic and static modelling of piezoelectric composite structures using a thermal analogy with MSC/NASTRAN," *Composite Structures*, vol. 65, no. 3-4, pp. 471-484, 2004.

[12] S. Herold, D. Mayer and H. Hanselka, "Transient Simulation of Adaptive Structures," *Journal of Intelligent Material Systems and Structures,* vol. 15, no. 3, pp. 215-224, 2004.

[13] W. Larbi, J.-F. De\"{u}, M. Ciminello and R. Ohayon, "Structural-Acoustic Vibration Reduction Using Switched Shunt Piezoelectric Patches: A Finite Element Analysis," *Journal of Vibration and Acoustics,* vol. 132, no. 5, p. 051006, 2010.

[14] F. Bachmann, A. E. Bergamini and P. Ermanni, "Optimum piezoelectric patch positioning: A strain energy–based finite element approach," *Journal of Intelligent Material Systems and Structures,* vol. 23, no. 14, pp. 1575-1591, 2012.

[15] A. Belloli and P. Ermanni, "Optimum placement of piezoelectric ceramic modules for vibration suppression of highly constrained structures," *Smart Materials and Structures,* vol. 16, no. 5, p. 1662, 2007.

[16] D. Saravanos, "Damped vibrations of composite plates with passive piezoelectric-resistor elements," *Journal of Sound and Vibration,* vol. 221, no. 5, pp. 867-885, 1999.

[17] T. S. Plagianakos and D. A. Saravanos, "Hybrid Multidamped Composite Plates with Viscoelastic Composite Plies and Shunted Piezoelectric Layers," *Journal of Intelligent Material Systems and Structures,* vol. 14, no. 1, pp. 57-66, 2003.

[18] C. Davis and G. Lesieutre, "A modal strain energy approach to the prediction of resistively shunted piezoceramic damping," *Journal of Sound and Vibration,* vol. 184, no. 1, pp. 129-139, 1995.

[19] T. Delpero, A. Bergamini and P. Ermanni, "Identification of electromechanical parameters in piezoelectric shunt damping and loss factor prediction," *Journal of intelligent material systems and structures,* vol. 24, no. 3, pp. 287-298, Feb 2013.

[20] M. Meier, "Optimierung von Flugzeugpropellerblättern für aerodynamische Untersuchungen im Windkanal," Master Thesis ETH Zurich, 2012.

[21] D. Cebon and N. F. Ashby, "Materials selection for precision instruments," *Measurement Science and Technology,* vol. 5, no. 3, p. 296, 1994.

[22] M. Ashby, Materials Selection in Mechanical Design, Elsevier Science, 2004.

[23] T. Delpero, "Design of adaptive structures with piezoelectric materials," Phd thesis Nr. 21839, ETH Zurich, 2014.